



School of Engineering
Department of Electrical and Computer Engineering

332:224 Principles of Electrical Engineering II Laboratory

Experiment 2

Frequency Response of Filters

1 Introduction

Objectives

- To introduce frequency response by studying the characteristics of two resonant circuits on either side of resonance

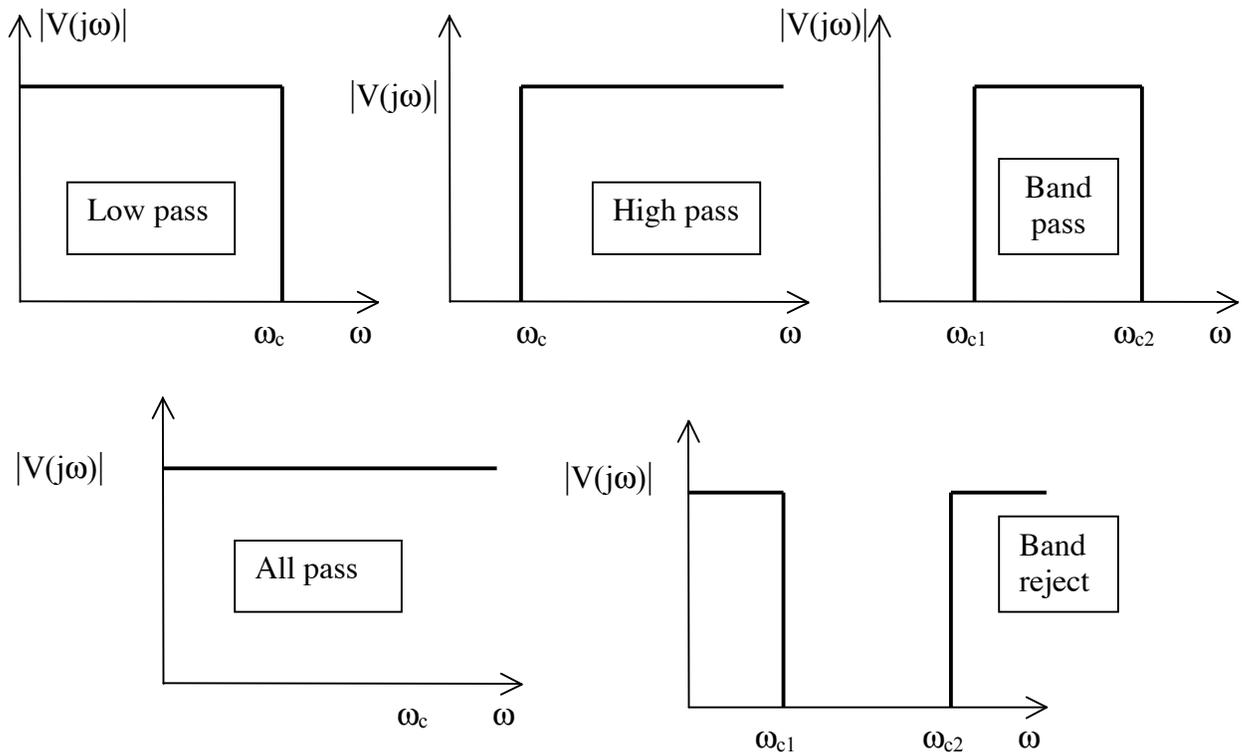
Overview

This experiment treats the subject of filters both in theory as well as with realized circuits. The Low Pass, High Pass, Band Pass, Band Reject and All Pass filters are introduced. These filters are characterized by their frequency response that indicates how near-ideal their filter operation actually is. Filters of different specifications are realized as mostly 2nd order active filters utilizing op-amps. Their frequency response (mostly the magnitude part but the phase part as well) is measured and the cut-off frequencies are determined.

2 Theory

2.1 Filters¹

An ideal filter is a network that allows signals of only certain frequencies to pass while blocking all others. Depending on the regime of frequencies that are allowed through or not they are characterized as *low-pass*, *high-pass*, *band-pass*, *band-reject* and *all-pass*. There are many needs for electric filters, some of the more common being those used in radio and television sets, which allow tuning in a certain channel by passing its band of frequencies while filtering out those of other channels. The *frequency response* is divided into *magnitude (amplitude)* and *phase* parts. The amplitude curve of a filter will indicate how closely the practical circuit imitates the ideal filter characteristics that are as follows:



Notice that, depending on the relative magnitude of their corner frequencies, a low pass in series with a high pass can either completely block all signals or give a band reject. By the same token, a low pass in parallel with a high pass can give either a bandpass or an all pass.

These ideal characteristics will at best be approximated by real circuits. How closely this will be achieved will depend on the frequency response of the particular circuit. The design of electric filters is based on a compromise between deviation from ideality vs. complexity (and cost). The order of the polynomial being used (thus the order of the filter) is the major

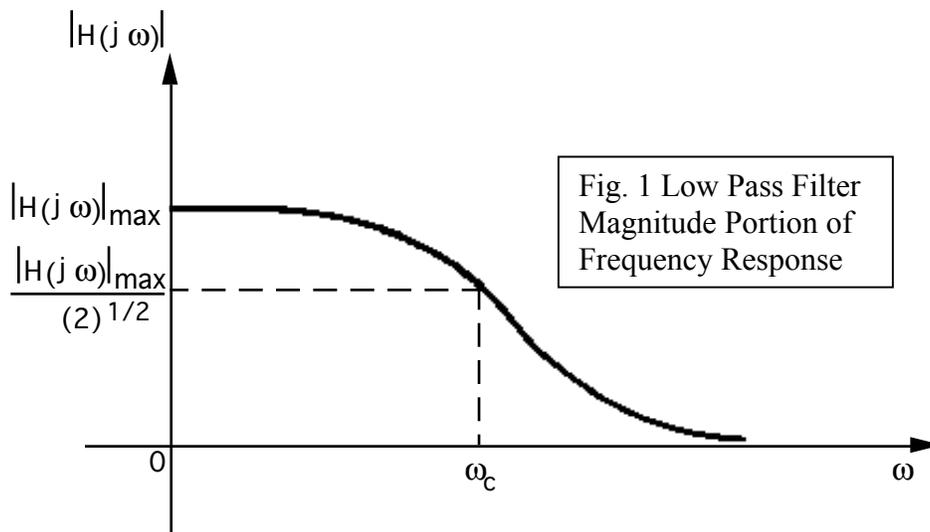
¹ The subject is treated in detail beginning with section 14.1 of the text.

indicator: the higher the order², the more complex the circuit and the closer the frequency response to the ideal curve. Filter operation is considered when the amplitude of the output signal of the circuit is relatively large for some frequencies and small for others. Then one can say that for the former the signal passes and for the latter it does not.

2.2 Practical Filters

2.2.1 Low Pass Filter

A low pass filter is a circuit whose amplitude (magnitude) function decreases as ω increases, that is, the circuit passes low frequencies (relatively large amplitudes at the output) and rejects high frequencies (relatively small amplitudes at the output) as shown in fig. 1.

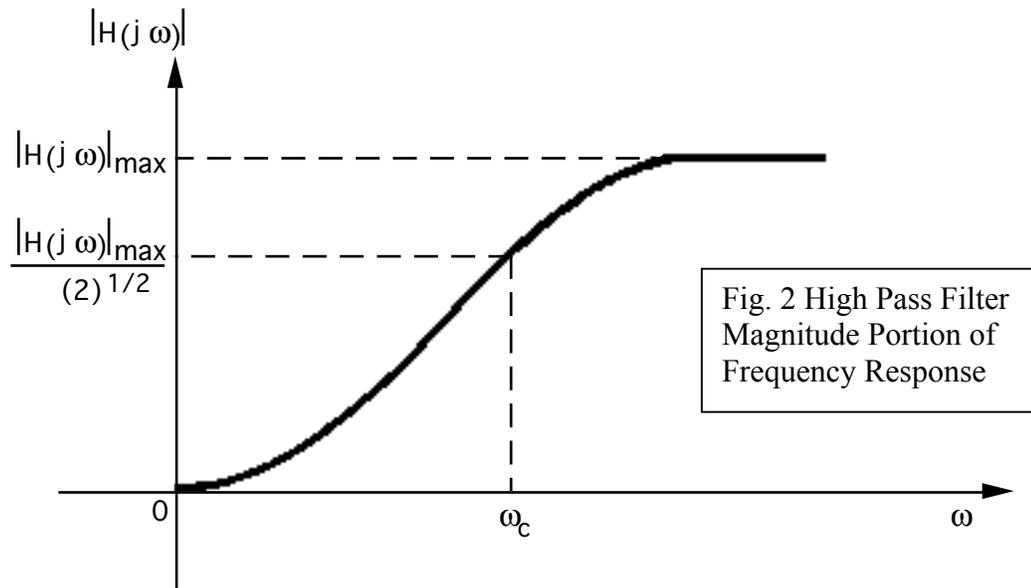


In fig. 1, ω_c is defined as the (3 dB) frequency, that is the frequency at which the amplitude is $(1/2)^{1/2} = 0.707$ times the maximum amplitude. It is traditional to consider the 3 dB frequency as the *cutoff frequency*. That is, a low pass filter is said to pass frequencies lower than ω_c and reject those that are higher than ω_c . In other words, the pass(ing) band is $\omega < \omega_c$.

2.2.2 High Pass Filter

A high pass filter is a circuit whose amplitude response increases with ω as shown in fig. 2.

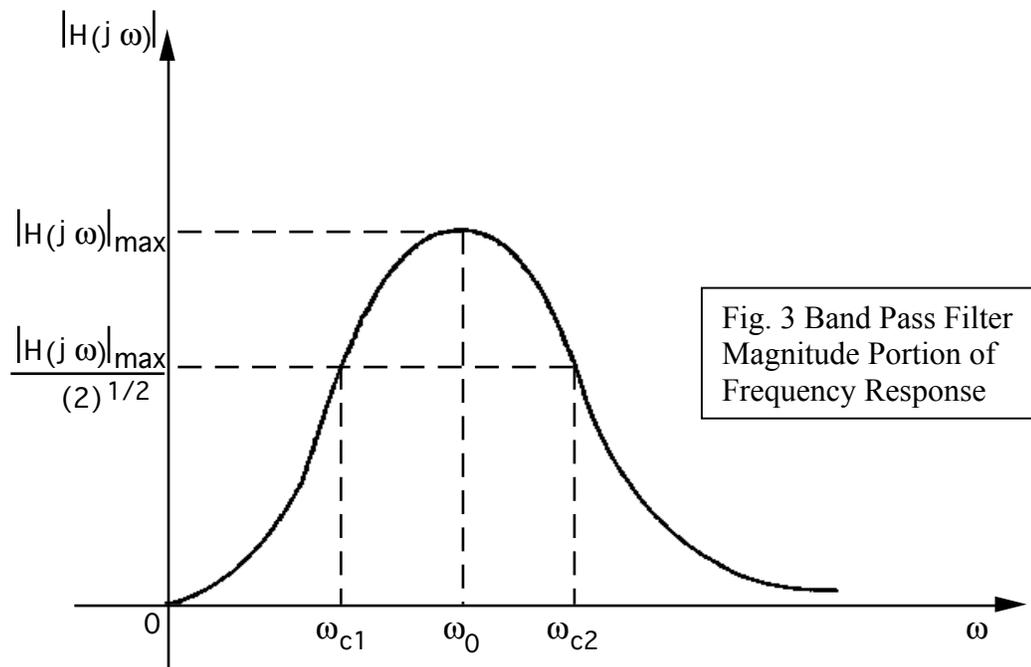
² The subject is treated in detail beginning with section 15.4 of the text.



This filter passes frequencies that are higher than the cutoff frequency ω_c and rejects those that are lower than ω_c . That is, the pass band is $\omega > \omega_c$.

2.2.3 Band Pass Filter

A band pass filter is a circuit which passes the band of frequencies centered around ω_0 as shown in fig. 3.

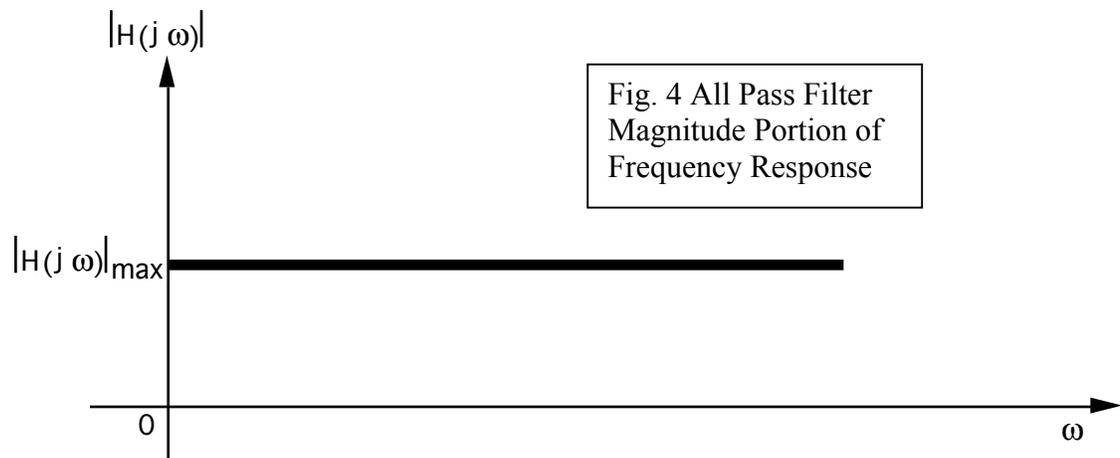


ω_0 is the frequency at which the maximum amplitude occurs, and is called the center frequency. The band of frequencies that passes, or the pass band, is defined to be $\omega_{c1} \leq \omega \leq \omega_{c2}$, where ω_{c1} and ω_{c2} are the cutoff (3 dB) frequencies.

The width of the pass band, given by $B = \omega_{c2} - \omega_{c1}$ is called the *bandwidth*.

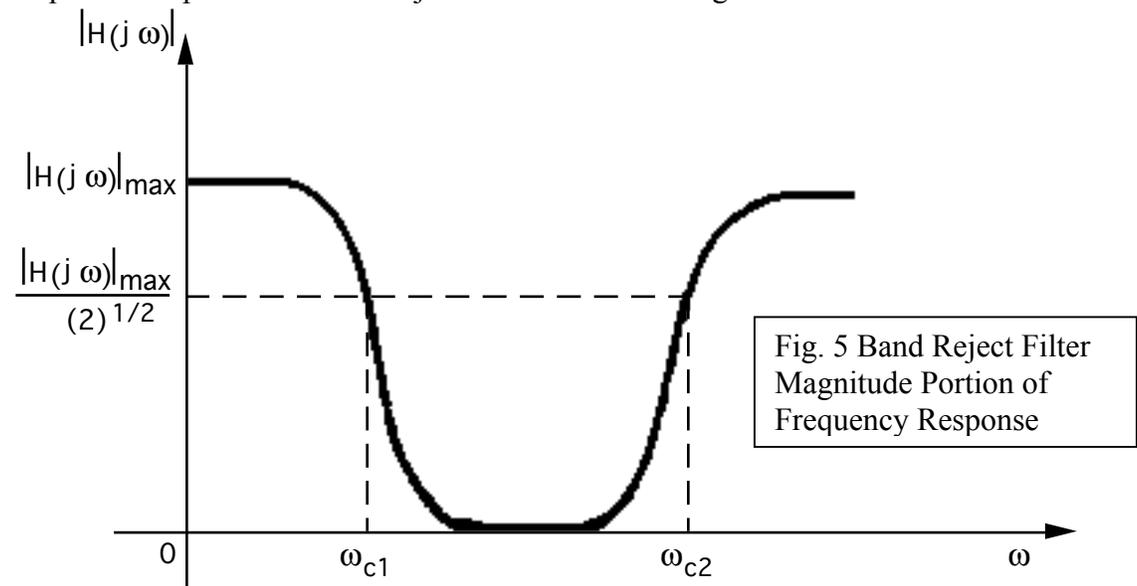
2.2.4 All Pass Filter

The all pass filter is one whose amplitude is constant, thus, it passes all frequencies equally well as shown in fig. 4. It is usually put in a cascade when it is desired to keep the amplitude part of the frequency response unaltered **but shift the phase** as desired.



2.2.5 Band Reject Filter

The band reject filter is one that passes all frequencies except a single band. A typical amplitude response of a band reject filter is shown in fig. 5.



As shown in fig. 5, the band reject filter passes all frequencies that are greater than ω_{c2} and smaller than ω_{c1} . Its stop band is $\omega_{c1} < \omega < \omega_{c2}$.

2.3 Review of Op Amps³

Filters realized with RLC circuits have limited performance. Filters incorporating such passive elements in addition to active elements such as op amps exhibit better characteristics and are called active filters⁴.

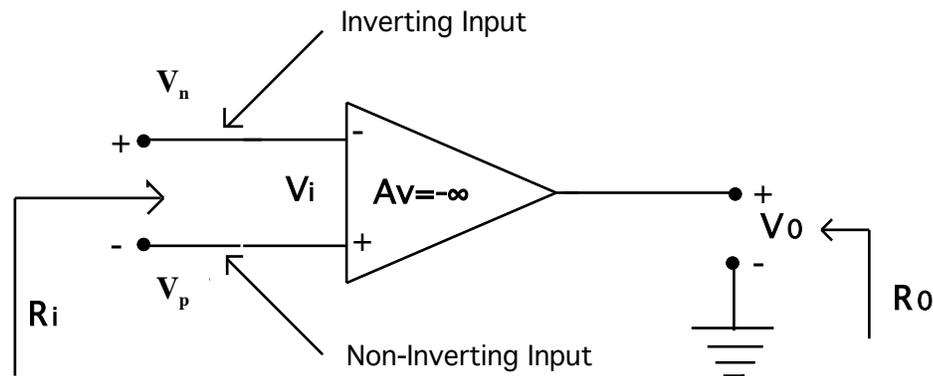


Fig. 1 Operational Amplifier

Characteristics of an Ideal Op Amp

1. Input Resistance $R_i = \infty$: An infinite input resistance means that no current flows into or out of either of the input terminals.
2. Output Resistance $R_o = 0$: In this case the output voltage V_o is independent of the output current
3. Open Loop Voltage Gain $\mu = A_V = \pm\infty$ ⁵:

In order to predict the behavior of an operational amplifier when circuit elements are externally connected to its terminals, one must understand the constraints imposed on the terminal voltages and currents by the amplifier itself. Those imposed on the terminal voltages are as follows :

$$V_o = A_V (V_p - V_n) \quad (1)$$

and

³ A somewhat more detailed description can be found in part 2 of Principles of EE I lab IV and a full description in sections 5.1-5.2 of the text.

⁴ The subject is treated in detail in chapter 15 of the text.

⁵ The sign of A_V (whether positive or negative) depends on the definition of V_i i.e. which of the two input terminals is defined as the reference. If V_i is defined as $(V_n - V_p)$ then A_V is <0 ; if defined as $(V_p - V_n)$ then it is >0 .

$$V = -V_{CC} \leq V_o \leq +V_{CC} = V^+ \quad (2)$$

Eq. 1 states that the output voltage is proportional to the difference between V_p and V_n . If the output voltage V_o is to be finite⁶ it follows from the definition of voltage gain, that $V_i = V_o / A_v$ will go to zero when A_v is infinite. This, however, assumes that there is some way for the input to be affected by the output. Indeed this will only happen if there is *negative feedback* in the form of a *connection between the output and the inverting terminal* (closed loop operation). For closed loop operation, it is said that a *virtual short* exists between the inverting and noninverting input terminals. This means that if an Op Amp is operating in its linear region (if it is *unsaturated*) then $V_i = 0$, or equivalently $V_n = V_p$.

Eq. 2 states that the output voltage is bounded. In particular, V_o must lie between $\pm V_{CC}$, the power supply voltages. Else V_o will be at either limiting value, and the Op-Amp is then saturated. The amplifier is operating in its linear range so long as $|V_o| < |V_{CC}|$.

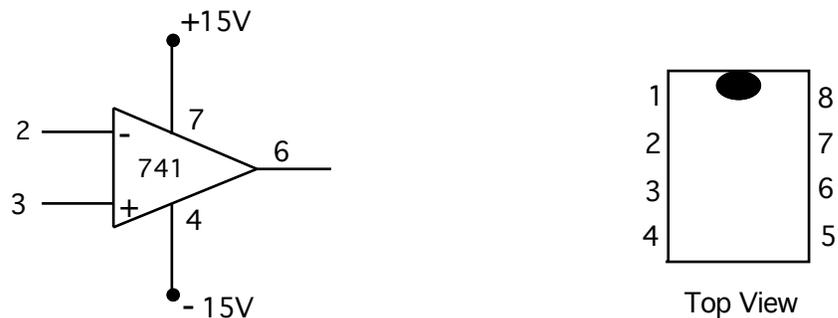


Fig. 2 741 OP-AMP

The chip layout is shown in Fig. 2. The standard procedure on a DIP is to identify pin 1 with the notch in the end of the chip package. The notch always separates pin 1 from the last pin on the chip. In the case of 741, the notch is between pins 1 and 8. Pins 2, 3, and 6 are the inverting input V_n , the non-inverting input V_p , and the amplifier output V_o respectively. These three pins are the three terminals that normally appear in an op-amp circuit schematic diagram. The null offset pins (1 and 5) provide a way to eliminate any offset in the output voltage of the amplifier. The offset voltage is an artifact of the integrated circuit. The offset voltage is additive with pin V_o (pin 6 in this case), can be either positive or negative and is normally less than 10 mV. Because of its small magnitude, in most cases, one can ignore the contribution of the offset voltage to V_o and leave the null offset pins open.

⁶ or rather unsaturated since when V_o tries to become larger than V^+ or smaller than V^- it gets clamped to V^+ or V^- respectively (or to a constant voltage somewhat less).

3 Prelab Exercises

3.1 Show why the cutoff frequency (or the frequency at which the output amplitude is $(1/2)^{1/2}$ times the maximum output amplitude) is called the 3 dB frequency.

3.2

- (a) Determine the amplitude response and the phase response functions for the low pass filter shown in fig. 7.
- (b) Determine the maximum amplitude of the output.
- (c) Determine the 3 dB frequency and its corresponding amplitude.

3.3 Repeat 3.2 for the high pass filter shown in fig. 8.

3.4 Repeat 3.2 for the band pass filter shown in fig. 9.

4 Experiments

Suggested Equipment:

Tektronix FG 501A 2MHz Function Generator⁷
 Tektronix PS 503 Power Supply
 Tektronix DC 504A Counter-Timer
 HP 54600A or Agilent 54622A Oscilloscope
 2: 620 Ω , 2: 1 K Ω , 2: 2K Ω , 2: 2.2K Ω , 20 K Ω Resistors
 0.05 μ F, 3: 0.1 μ F, 0.01 μ F Capacitors
 741 Op-Amp
 Protoboard

4.1 Low Pass Filter

Build the low pass filter circuit shown in fig. 7. Apply a sinusoidal input of amplitude 1V rms and display both input and output on the scope.

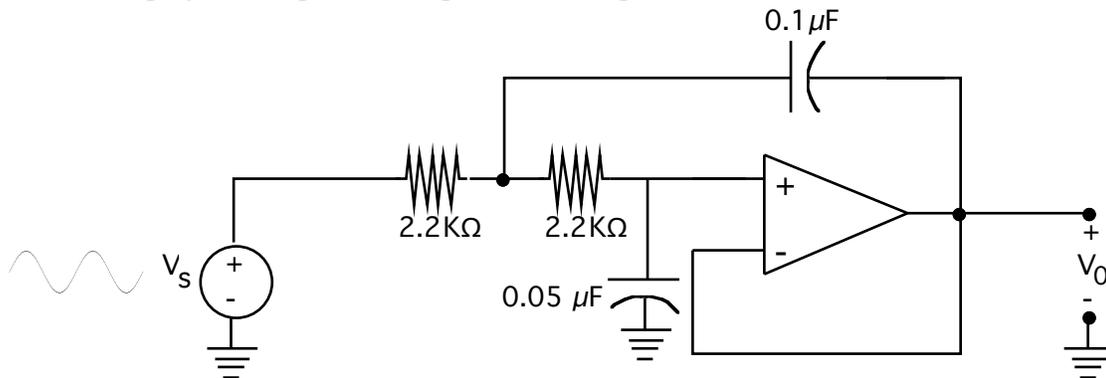


Fig. 7 A Second Order Low Pass Filter

Vary the frequency of the input voltage from 100 Hz to 3000 Hz, and measure the rms output voltage and the phase angle⁸. Take as many readings as are necessary to develop a well-defined plot. Make sure to set $V_s = 1$ V rms before each reading.

Find the exact cutoff (3 dB) frequency and the exact corresponding output voltage V_0 .

⁷ NOTE: The oscillator is designed to work for a very wide range of frequencies but may not be stable at very low frequencies, say in the order of 100 Hz or 200Hz. To start with it is a good idea to have the circuit working at some mid-range frequency, say in the order of 1K Hz or 2K Hz, and then change the frequency slowly as needed.

⁸ The phase angle between two sinusoidal signals of the same frequency can be determined as follows: Trace both signals on two different channels with the same horizontal sensitivities (the same horizontal scale). To calibrate the horizontal scale in terms of degrees, one can use the fact that the angular difference between the two successive zero crossing points of a sinusoidal signal is 180 degrees. Thus, by measuring the distance between the successive zero crossing points of either sinusoidal signal, one can calibrate the horizontal scale in terms of degrees. To determine the phase difference between the two sinusoidal signals, determine the distance between the zero crossing point of one signal to a similar zero crossing point of another signal and convert it into degrees.

4.4 Band Reject Filter (Optional)

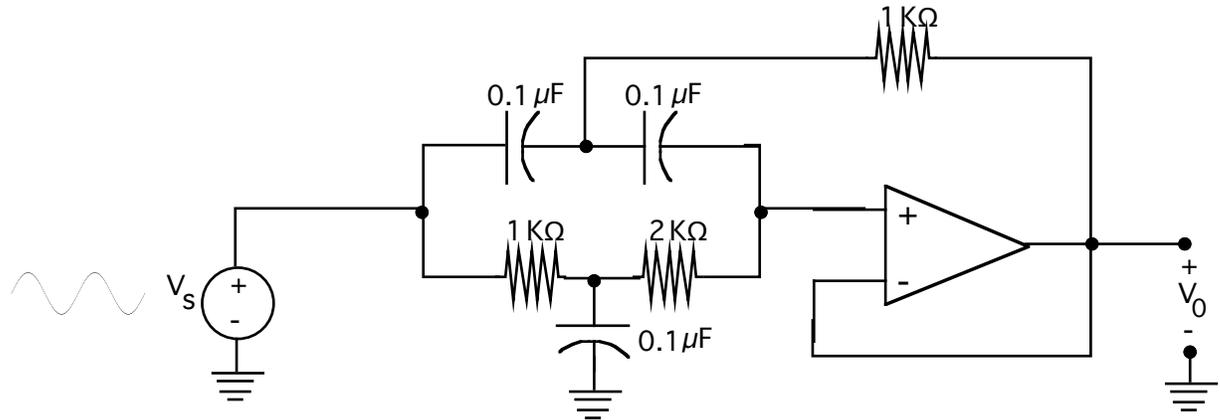


Fig. 11 A Third Order Band Reject Filter

Build the band reject filter circuit shown in fig. 11.

- Show that the circuit is indeed a band reject filter.
- Determine the maximum amplitude of the output.
- Determine the minimum amplitude of the output and its corresponding frequency.
- Determine the 3 dB frequencies and their corresponding output amplitudes.
- Determine the stopband.
- Read some phase angles at some particular frequencies.

4.5 All Pass Filter (optional)

Build the all pass filter circuit shown in fig. 10. Take readings of the output voltage and the phase with the frequency varied from 100 Hz to 10 KHz.

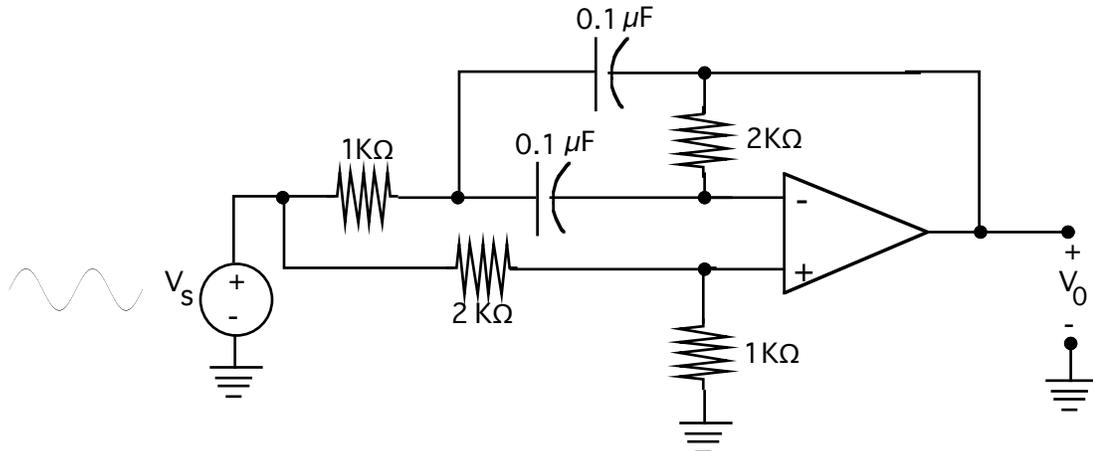


Fig. 10 A Second Order All Pass Filter

5 Report

- 5.1** For the low-pass filter, using the functions determined in pre-lab exercise 3.2 (a), determine the theoretical values of the amplitude and the phase at the frequencies that were used to take readings during the experiment. Tabulate the theoretical and the experimental values of both amplitude and phase. On graph paper with rectangular coordinates, plot the theoretical and the experimental data. Compare the theoretical plot with the experimental one.
- 5.2** For the high-pass filter, repeat 5.1 using the functions determined in pre-lab exercise 3.3 (a).
- 5.3** For the band-pass filter, repeat 5.1 using the functions determined in pre-lab exercise 3.4 (a).
- 5.4** Simulate, in PSpice, the low pass filter circuit in fig. 7, and plot the amplitude and the phase response in the interval $0.001 \text{ Hz} \leq f \leq 2000 \text{ Hz}$.
- 5.5** Simulate in PSpice, the high-pass and band pass filter circuits of figures 8 and 9 in appropriate frequency ranges.
- 5.6** Prepare a summary.