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332:580 – Electric Waves and Radiation
Exam 1 – October 8, 1997

1. A Doppler radar for measuring the speed of a vehicle may be modeled as a uniform plane wave incident normally on a perfectly conducting surface which is moving away from the source with a speed v.

   By matching the boundary conditions at the moving conducting surface, derive an expression for the Doppler frequency shift.

   If the incident wave has frequency 9 GHz and the measured Doppler shift is $\Delta f = 2$ kHz, determine the vehicle's speed in km/hr.

2. A left-hand polarized plane wave represented by the phasor

$$\mathbf{E}(z,t) = E_0\,(\hat{\mathbf{x}} + j\hat{\mathbf{y}})\,e^{j\omega t - jkz}$$

   is normally incident from free space on a perfectly conducting wall at $z = 0$. Determine the polarization of the reflected wave.

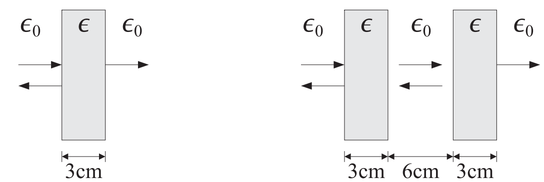
3. A uniform plane wave of frequency of 1.25 GHz is normally incident from free space onto a fiberglass dielectric slab ($\epsilon = 4\epsilon_0, \mu = \mu_0$) of thickness of 3 cm, as shown on the left figure below.

   (a) What is the free-space wavelength of this wave in cm? What is its wavelength inside the fiberglass?

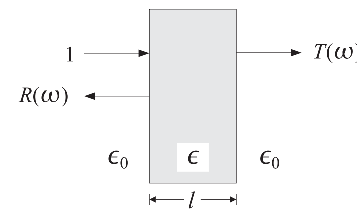
   (b) What percentage of the incident power is reflected backwards?

   Next, an identical slab is inserted to the right of the first slab at a distance of 6 cm, as shown on the right figure below.

   (c) What percentage of incident power is now reflected back?



4. A uniform plane wave of frequency ω is normally incident from the left on a lossless dielectric slab ϵ of thickness l. We may assume that the medium to the left and to the right of the slab is air.



   Let $R(\omega)$ and $T(\omega)$ be the reflection response into the left and the transmission response to the right, as shown. Determine expressions of $R(\omega)$ and $T(\omega)$ as functions of frequency, and then show that they satisfy the relationship:

$$|R(\omega)|^2 + |T(\omega)|^2 = 1$$

   What does this relationship imply about energy conservation?

Hints

$$\begin{bmatrix} E_{1+} \\ E_{1-} \end{bmatrix} = \frac{1}{\tau}\begin{bmatrix} 1 & \Gamma \\ \Gamma & 1 \end{bmatrix}\begin{bmatrix} e^{jkl} & 0 \\ 0 & e^{-jkl} \end{bmatrix}\begin{bmatrix} E_{2+} \\ E_{2-} \end{bmatrix}$$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}, \qquad \tau = 1 + \Gamma$$

$$Z_{\text{in}} = \eta\,\frac{\eta_{\text{load}}\cos kl + j\eta\sin kl}{\eta\cos kl + j\eta_{\text{load}}\sin kl}$$

332:580 – Electric Waves and Radiation
Exam 2 – November 10, 1997

1. A unform plane wave is incident from free-space onto a planar dielectric at an angle θ. The dielectric is non-magnetic and has refractive index n. Let θ_t be the refracted angle into the dielectric. The reflection coefficient for *parallel polarization* is given as follows:

$$\Gamma = \frac{n^2\cos\theta - \sqrt{n^2 - \sin^2\theta}}{n^2\cos\theta + \sqrt{n^2 - \sin^2\theta}}$$

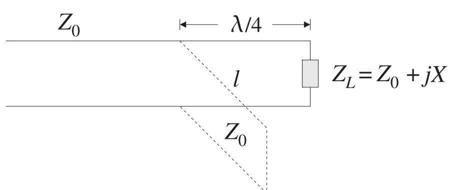
   (a) Using Snell's law, show that Γ can be re-expressed in the equivalent forms:

$$\Gamma = \frac{n\cos\theta - \cos\theta_t}{n\cos\theta + \cos\theta_t} = \frac{\sin(2\theta) - \sin(2\theta_t)}{\sin(2\theta) + \sin(2\theta_t)} = \frac{\tan(\theta - \theta_t)}{\tan(\theta + \theta_t)}$$

   (b) Determine the expression, $\tan\theta_B = n$, for the Brewster angle by requiring the condition $\theta + \theta_t = 90^{\circ}$.

2. A loss-free line of impedance Z_0 is terminated at a load $Z_L = Z_0 + jX$, which is not quite matched to the line. To properly match the line, a short-circuited stub is connected across the main line at a distance of $\lambda/4$ from the load, as shown below. The stub has characteristic impedance Z_0.

   Find an equation that determines the length l of the stub in order that there be no reflected waves into the main line. What is the length l (in wavelengths λ) when $X = Z_0$? When $X = Z_0/\sqrt{3}$?



3. A 100-Ω lossless transmission line is terminated at an unknown load impedance. The line is operated at a frequency corresponding to a wavelength $\lambda = 40$ cm.

   The standing wave ratio along this line is measured to be $S = 3$. The distance from the load where there is a voltage minimum is measured to be 5 cm. Based on these two measurements, determine the unkown load impedance. [*Hint:* First determine Γ and note $\Gamma = |\Gamma|e^{j\theta}$.]

4. A TE_{10} mode of frequency $\omega > \omega_c$ is propagated along a rectangular waveguide of dimensions a, b. The longitudinal magnetic field is

$$H_z = H_0\cos\left(\frac{\pi x}{a}\right)$$

   (a) Determine expressions of the remaining field components E_y and H_x.

   (b) Determine expressions (in terms of H_0, ω, a, b, etc.) for the time-averaged power P transmitted down the guide, and for the electric and magnetic energy densities per unit z-length, U'_{el} and U'_{mag}.

   (c) Show that $U'_{\text{el}} = U'_{\text{mag}}$. And determine the total $U' = U'_{\text{el}} + U'_{\text{mag}}$.

   (d) Show that the velocity by which energy is propagated down the guide is equal to the group velocity, that is, show

$$\frac{P}{U'} = v_g = c\sqrt{1 - \frac{\omega_c^2}{\omega^2}}$$

Hints

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \sin\beta\cos\alpha\,, \qquad \cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

$$\sin\alpha\cos\beta = \frac{1}{2}\left[\sin(\alpha+\beta) + \sin(\alpha-\beta)\right], \qquad \sin(2\alpha) = 2\sin\alpha\cos\alpha$$

$$V(l) = V_L\frac{1 + \Gamma e^{-2j\beta l}}{1+\Gamma}\,, \qquad S = \frac{1+|\Gamma|}{1-|\Gamma|}\,, \qquad Z_{\text{in}} = Z_0\frac{Z_L + jZ_0\tan(\beta l)}{Z_0 + jZ_L\tan(\beta l)}$$

$$\mathbf{H}_T = -\frac{j\beta}{h^2}\nabla_T H_z\,, \qquad \mathbf{E}_T = Z_{TE}\,\mathbf{H}_T\times\hat{\mathbf{z}}\,, \qquad Z_{TE} = \frac{\eta}{\sqrt{1 - \frac{\omega_c^2}{\omega^2}}}$$

332:580 – Electric Waves and Radiation
Final Exam – December 16, 1997

1. A transmission line with characteristic impedance Z_0 must be matched to a purely resistive load Z_L. A segment of length l_1 of another line of characteristic impedance Z_1 is inserted at a distance l_0 from the load, as shown below.

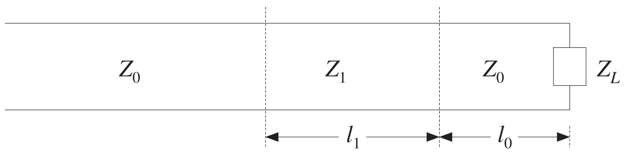
   Take $Z_0 = 50$, $Z_1 = 100$, $Z_L = 80\,\Omega$ and let β_0 and β_1 be the wavenumbers within the segments l_0 and l_1.

   (a) Determine the values of the quantities $\cot(\beta_1 l_1)$ and $\cot(\beta_0 l_0)$ that would guarantee matching.

   (b) Not all possible resistive loads Z_L can be matched by this method. Show that the widest range of Z_L that can be matched using the given values of Z_0 and Z_1 is:

$$12.5\ \Omega < Z_L < 200\ \Omega$$

   [*Hint:* $Z_0 \dfrac{Z_L + jZ_0\tan(\beta l)}{Z_0 + jZ_L\tan(\beta l)}$. Work with normalized impedances.]



2. An antenna is transmitting power P_T with gain G_T. A receiving antenna at a distance r has gain G_R. Let λ be the operating wavelength. Assuming that the two antennas are oriented towards the maximal gain of each other, show that the received power is given by

$$P_R = P_T G_T G_R G_F$$

   where $G_F = (\lambda/4\pi r)^2$ is the free-space "gain". Be sure to explain carefully where each factor comes from.

3. Consider a mobile radio channel in which the transmitting antenna at the base station is at height h_1 from the ground and the receiving mobile antenna is at height h_2, as shown below. The ray reflected from the ground interferes with the direct ray and can cause substantial signal cancellation at the receiving antenna.

   Let r be the distance from the origin O to the receiving antenna R. You may assume that h_1 is in the z-direction.

   The reflected ray may be thought of as originating from the *image* of the transmitting antenna at $-h_1$, as shown. Thus, we have an equivalent two-element transmitting array. We assume that the currents on the actual and image antennas are $I(z)$ and $\Gamma I(z)$, where Γ is the reflection coefficient of the ground. It can be shown that for near-grazing angles of incidence for the reflected wave, $\Gamma \simeq -1$.

   (a) Assuming that Γ is real-valued, determine the array factor $A(\theta)$ and show that its magnitude square can be written in the form:

$$G_A(\theta) = |A(\theta)|^2 = (1+\Gamma)^2 - 4\Gamma\sin^2(kh_1\cos\theta)$$

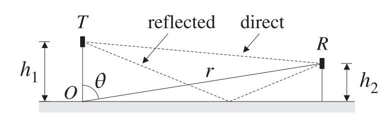
   where $k = 2\pi/\lambda$. Thus, the gain of the transmitting antenna is effectively changed into $G_T \to G_T G_A$, and therefore, the received power will be: $P_R = P_T G_T G_R G_F G_A$.

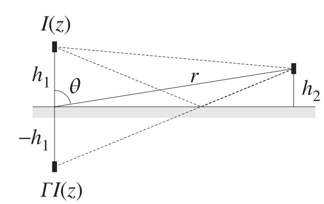
   (b) Assuming $r^2 \gg h_1 h_2$ and $\Gamma = -1$, show that the received power of part (a) takes the approximate form:

$$P_R = P_T G_T G_R \frac{h_1^2 h_2^2}{r^4}$$

   Thus, it is falls like $1/r^4$, instead of the usual $1/r^2$.

   (c) Assuming $f = 800$ MHz, $h_1 = 100$ ft, $h_2 = 6$ ft, and $r = 3$ mi, determine by how many dB the received power will be smaller as compared to the power that would be received if there were no ground reflections at all.





332:580 – Electric Waves and Radiation
Exam 1 – October 12, 1998

1. Determine the polarization types of the following plane waves:

$$\mathbf{E}(z) = E_0(2\hat{\mathbf{x}} + j\hat{\mathbf{y}})\,e^{-jkz}$$
$$\mathbf{E}(z) = E_0(j\hat{\mathbf{x}} + 2\hat{\mathbf{y}})\,e^{+jkz}$$

Express the first case as a linear combination of a left and a right circularly polarized wave.

2. A uniform plane wave is obliquely incident from air onto a lossless dielectric with refractive index n. Assuming perpendicular polarization, the incident, reflected, and transmitted electric fields are given by:

$$\mathbf{E} = \hat{\mathbf{y}}\,E_y = E_0\,\hat{\mathbf{y}}\,e^{-jkx\sin\theta - jkz\cos\theta}$$
$$\mathbf{E}'' = \hat{\mathbf{y}}\,E_y'' = \rho E_0\,\hat{\mathbf{y}}\,e^{-jkx\sin\theta + jkz\cos\theta}$$
$$\mathbf{E}' = \hat{\mathbf{y}}\,E_y' = \tau E_0\,\hat{\mathbf{y}}\,e^{-jk'x\sin\theta' - jk'z\cos\theta'}$$

where ρ, τ are the reflection and transmission coefficients, θ, θ' are the incident and refracted angles, and $k' = nk = n\omega/c_0$.

(a) Derive the following expressions for the z-components of the time-averaged Poynting vectors for the transmitted, reflected, and refracted waves:

$$\mathcal{P}_z = \frac{|E_0|^2}{2\eta_0}\cos\theta\,, \quad \mathcal{P}_z'' = -\frac{|\rho E_0|^2}{2\eta_0}\cos\theta\,, \quad \mathcal{P}_z' = \frac{|\tau E_0|^2}{2\eta_0}\,n\cos\theta'$$

[*Hints:* Identify $\hat{\mathbf{k}}, \hat{\mathbf{k}}'', \hat{\mathbf{k}}'$. Recall that $\boldsymbol{\mathcal{P}} = \hat{\mathbf{k}}\,|\mathbf{E}|^2/2\eta_0$.]

(b) Assume the interface coincides with the xy plane at $z = 0$. The following two conditions express the continuity of the tangential electric field and energy flux in the z direction:

$$E_y + E_y'' = E_y'$$
$$\mathcal{P}_z + \mathcal{P}_z'' = \mathcal{P}_z'$$

Using these two conditions, derive Snell's law, $\sin\theta = n\sin\theta'$, and the following expression for the reflection coefficient:

$$\rho = \frac{\cos\theta - \sqrt{n^2 - \sin^2\theta}}{\cos\theta + \sqrt{n^2 - \sin^2\theta}}$$

3. A plane wave is incident at an angle θ onto a planar interface separating a lossless dielectric of refractive index n and air. The wave is incident from the *inside* of the dielectric. The reflection coefficients for perpendicular and parallel polarizations are given by:

$$\rho_\perp = \frac{\cos\theta - \sqrt{n^{-2} - \sin^2\theta}}{\cos\theta + \sqrt{n^{-2} - \sin^2\theta}}\,, \quad \rho_\parallel = \frac{n^{-2}\cos\theta - \sqrt{n^{-2} - \sin^2\theta}}{n^{-2}\cos\theta + \sqrt{n^{-2} - \sin^2\theta}}$$

(a) Using these expressions, show that the Brewster angle and the critical angle for total internal reflection are given by $\tan\theta_B = 1/n$ and $\sin\theta_c = 1/n$.

(b) Show mathematically that always $\theta_B < \theta_c$, so that the Brewster angle never corresponds to total internal reflection.

(c) Sketch a plot of the power reflection coefficients $|\rho_\perp|^2$ and $|\rho_\parallel|^2$ versus the incident angle θ in the range $0 \le \theta \le 90^o$. Indicate the angles θ_B and θ_c on your graph.

(d) When the incident angle is equal to the Brewster angle, show that the perpendicular reflection coefficient is given by:

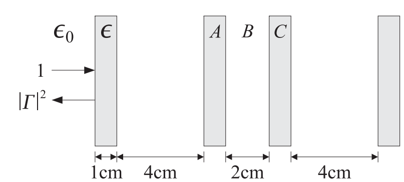
$$\rho_\perp = \frac{n^2 - 1}{n^2 + 1}$$

4. Four identical dielectric slabs of thickness of 1 cm and dielectric constant $\epsilon = 4\epsilon_0$ are positioned as shown below. A uniform plane wave of frequency of 3.75 GHz is incident normally onto the leftmost slab.

(a) Determine the power reflection coefficient $|\Gamma|^2$ as a percentage.

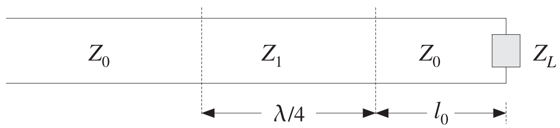
(b) Determine $|\Gamma|^2$ if slabs A and C are removed and replaced by air.

(c) Determine $|\Gamma|^2$ if the air gap B between slabs A and C is filled with the same dielectric, so that ABC is a single slab.



332:580 – Electric Waves and Radiation
Exam 2 – November 23, 1998

1. It is required to match a lossless transmission line Z_0 to a load Z_L. To this end, a quarter-wavelength transformer is connected at a distance l_0 from the load, as shown below. Let λ_0 and λ be the operating wavelengths of the line and the transformer segment.



Assume $Z_0 = 50\ \Omega$. Verify that the required length l_0 that will match the complex load $Z_L = 40 + 30j\ \ \Omega$ is $l_0 = \lambda/8$. What is the value of Z_1 in this case?

2. The wavelength on a 50 Ω transmission line is 80 cm. Determine the load impedance if the SWR on the line is 3 and the location of the first voltage minimum is 10 cm from the load.

   At what other distances from the load would one measure a voltage minimum? A voltage maximum?

3. A TE_{10} mode of frequency ω is propagated along an air-filled rectangular waveguide of sides a and b. Let $\omega_c = \pi c/a$ and $h = \pi/a$ be the cutoff frequency and cutoff wavenumber. The non-zero field components are given by (the $e^{-j\beta z}$ factor is not shown):

$$E_y = E_0 \sin(hx), \quad H_x = H_1 \sin(hx), \quad H_z = H_0 \cos(hx)$$

   (a) Derive the relationship of the constants H_1, H_0 to E_0.

   (b) By integrating the time-averaged volume energy densities over the cross-sectional area of the guide, show that the *electric* and *magnetic* energy densities per unit length along the guide are given by:

$$W'_e = \frac{1}{8} ab\,\epsilon |E_0|^2, \qquad W'_m = \frac{1}{8} ab\,\mu\left(|H_0|^2 + |H_1|^2\right)$$

   (c) Show that $W'_e = W'_m$. Let $W' = W'_e + W'_m = 2W'_e$ be the total energy density per unit length. By multiplying W' by the group velocity $v_g = c\sqrt{1 - \omega_c^2/\omega^2}$, show that the total power transmitted down the guide is given by

$$P = \frac{1}{4\eta}\, ab\, |E_0|^2 \sqrt{1 - \omega_c^2/\omega^2}$$

4. Explain why an "optimal" rectangular waveguide must have sides $b = a/2$ and must be operated at a frequency $f = 1.5 f_c$, where f_c is the minimum cutoff frequency.

5. An air-filled rectangular waveguide is used to transfer power to a radar antenna. The guide must meet the following specifications:

   The two lowest modes are TE_{10} and TE_{20}. The operating frequency is 3 GHz and must lie exactly halfway between the cutoff frequencies of these two modes. The maximum electric field within the guide may not exceed, by a safety margin of 3, the breakdown field of air 3 MV/m.

   (a) Determine the smallest dimensions a, b for such a waveguide, if the transmitted power is required to be 1 MW.

   (b) What are the dimensions a, b if the transmitted power is required to be maximum? What is that maximum power in MW?

Hints

$$Z = Z_0 \frac{Z_L + jZ_0 \tan kl}{Z_0 + jZ_L \tan kl}$$

$$\int_0^a \sin^2(\pi x/a)\,dx = \int_0^a \cos^2(\pi x/a)\,dx = \frac{a}{2}$$

$$c\epsilon = \frac{1}{\eta}, \quad f_c = \frac{c}{2}\sqrt{\frac{m^2}{a^2} + \frac{n^2}{b^2}}$$

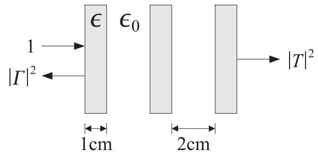
$$H_x = -\frac{j\beta}{h^2}\,\partial_x H_z, \quad E_y = \frac{j\omega\mu}{h^2}\,\partial_x H_z$$

332:580 – Electric Waves and Radiation
Final Exam – December 16, 1998

1. Three identical dielectric slabs of thickness of 1 cm and dielectric constant $\epsilon = 4\epsilon_0$ are positioned as shown below. A uniform plane wave of frequency of 3.75 GHz is incident normally onto the leftmost slab.

   (a) Determine the power reflection and transmission coefficients, $|\Gamma|^2$ and $|T|^2$, as percentages of the incident power.

   (b) Determine $|\Gamma|^2$ and $|T|^2$ if the three slabs and air gaps are replaced by a single slab of thickness of 7 cm.



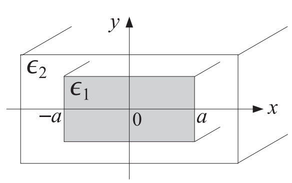
2. A TE mode of frequency ω and wavenumber β is propagated along the z-direction in a rectangular dielectric waveguide with refractive index n_1. The waveguide is surrounded by a cladding material of refractive index $n_2 < n_1$, as shown below.

   The longitudinal magnetic field component is given in the regions inside and outside the guide as follows:

$$H_z(x) = \begin{cases} H_1 \sin(h_1 x), & -a \le x \le a \\ H_2\, e^{-\alpha_2 x}, & x \ge a \\ -H_2\, e^{\alpha_2 x}, & x \le -a \end{cases}$$

   (a) Determine similar expressions for the remaining field components. Determine the relationship of h_1, α_2 to ω, β.

   (b) Applying the proper boundary conditions, determine the relationship between the constants H_1, H_2 and the relationship between β and ω.



3. A Hertzian dipole antenna has normalized power gain $g(\theta) = \sin^2\theta$. Determine the 3-dB beam width of this antenna by setting up and solving the defining conditions for this width.

4. In an earth-satellite-earth communication system, the uplink/downlink distances are 36000 km. The uplink/downlink frequencies are 6/4 GHz. The diameters of the earth and satellite antennas are 15 m and 0.5 m with 60% aperture efficiencies. The transmitting earth antenna transmits power of 1 kW. The satellite transponder gain is 90 dB. The satellite receiving antenna is looking down at an earth temperature of 300°K and has a noisy receiver of effective noise temperature of 2700°K, whereas the earth receiving antenna is looking up at a sky temperature of 50°K and uses a high-gain LNA amplifier of noise temperature of 80°K (feedline losses may be ignored.) The bandwidth is 30 MHz.

   (a) Calculate all antenna gains in dB.

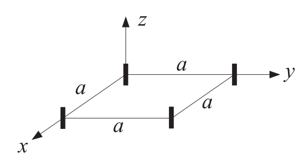
   (b) Calculate the uplink and downlink free-space losses in dB.

   (c) Calculate the amount of power received by the satellite in dBW. Calculate the uplink signal to noise ratio in dB.

   (d) Calculate the power received by the receiving earth antenna in dBW and the downlink signal to noise ratio.

   (e) Finally, calculate the total system signal to noise ratio in dB.

5. Four identical isotropic antennas are positioned on the xy-plane at the four corners of a square of sides a, as shown below. Determine the array factor $A(\phi)$ of this arrangement as a function of the azimuthal angle ϕ. (Assume the look direction is on the xy-plane.)



332:580 – Electric Waves and Radiation
Exam 1 – October 13, 1999

1. We construct a makeshift antenna by wrapping aluminum foil around a stick of wood. Aluminum foil has thickness of about 1/000 of an inch and conductivity 3.5×10^7 S/m. The antenna will operate adequately if the foil thickness is at least five skin depths at the operating frequency.

   Will such an antenna be adequate for UHF reception at 900 MHz? For VHF reception at 100 MHz? Do we need to wrap the foil around several times?

   Can the VHF case be answered quickly based on the answer for the UHF case, without having to recalculate everything?

   [*Hints:* $\alpha = \sqrt{\pi f \mu \sigma}$.]

2. Recent measurements (ca.1997) of the absorption coefficient α of water over the visible spectrum show that it starts at about 0.01 nepers/m at 380 nm (violet) and decreases to a minimum value of 0.0044 nepers/m at 418 nm (blue) and then increases steadily reaching the value of 0.5 nepers/m at 600 nm (red).

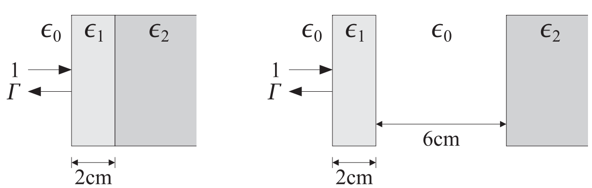
   For each of the three wavelengths, determine the depth in meters at which the light intensity has decreased to 1/10th its value at the surface of the water.

   [*Hint:* 8.8686 dB per delta.]

3. A 2.5 GHz wave is normally incident from air onto a dielectric slab of thickness of 2 cm and refractive index of 1.5, as shown below. The medium to the right of the slab has an index of 2.25.

   (a) Derive an analytical expression of the reflectance $|\Gamma(f)|^2$ as a function of frequency and sketch it versus f over the interval $0 \le f \le 10$ GHz. What is the value of the reflectance at 2.5 GHz?

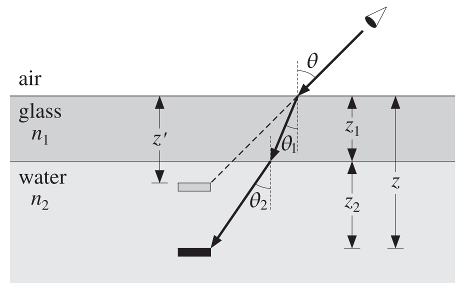
   (b) Next, the 2-cm slab is moved to the left by a distance of 6 cm, creating an air-gap between it and the rightmost dielectric. What is the value of the reflectance at 2.5 GHz?



4. An underwater object is viewed from air at an angle θ through a glass plate, as shown below. Let $z = z_1 + z_2$ be the actual depth of the object from the air surface, where z_1 is the thickness of the glass plate, and let n_1, n_2 be the refractive indices of the glass and water. Show that the apparent depth of the object is given by:

$$z' = \frac{z_1 \cos\theta}{\sqrt{n_1^2 - \sin^2\theta}} + \frac{z_2 \cos\theta}{\sqrt{n_2^2 - \sin^2\theta}}$$

[*Hint:* $x = x_1 + x_2$.]



332:580 – Electric Waves and Radiation
Exam 2 – November 22, 1999

1. It is desired to design an air-filled rectangular waveguide operating at 5 GHz, whose group velocity is $0.8c$. What are the dimensions a, b of the guide (in cm) if it is also required to carry maximum power and have the widest bandwidth possible? What is the cutoff frequency of the guide in GHz and the operating bandwidth?

2. Show the following relationship between guide wavelength and group velocity in an arbitrary air-filled waveguide:

$$v_g \lambda_g = c\lambda$$

where $\lambda_g = 2\pi/\beta$ and λ is the free-space wavelength.

3. A 75-ohm line is connected to an unknown load. Voltage measurements along the line reveal that the maximum and minimum voltage values are 6 V and 2 V. It is observed that a voltage maximum occurs at the distance from the load:

$$l = 0.5\lambda - \frac{\lambda}{4\pi}\text{atan}(0.75) = 0.44879\lambda$$

Determine the reflection coefficient Γ_L (in cartesian form) and the load impedance Z_L.

4. The Arecibo Observatory in Puerto Rico has a gigantic dish antenna of diameter of 1000 ft (304.8 m). It transmits power of 2.5 MW at a frequency of 430 MHz.

   (a) Assuming a 60 percent effective area, what is its gain in dB?

   (b) What is its beamwidth in degrees?

   (c) If used as a radar and the minimum detectable received power is -130 dBW, what is its maximum range for detecting a target of radar cross-section of 1 m^2?

5. For a highly directive antenna, show that the relationship between the directivity D and the solid angle $\Delta\Omega$ subtended by the beam is given by

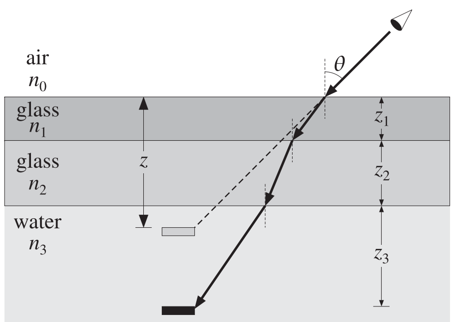
$$D = \frac{4\pi}{\Delta\Omega}$$

State the assumptions and approximations that are necessary to derive this expression.

332:580 – Electric Waves and Radiation
Final Exam – December 17, 1999

1. An underwater object is viewed from air at an angle θ through two glass plates of refractive indices n_1, n_2 and thicknesses z_1, z_2, as shown below. Let z_3 be the depth of the object within the water.

   Express the apparent depth z of the object in terms of the quantities θ, n_0, n_1, n_2, n_3 and z_1, z_2, z_3.



2. A conducting waveguide has a triangular cross section as shown below. A TM mode has E_z field component given by $E_z(x,y,)e^{j\omega t - j\beta z}$, where

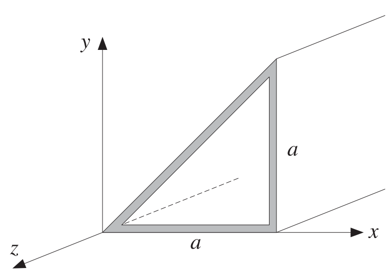
$$E_z(x,y) = E_0(\sin k_1 x \sin k_2 y - \sin k_2 x \sin k_1 y)$$

   (a) Derive the relationship among the quantities ω, β, k_1, k_2.

   (b) Determine the remaining E-field components $E_x(x,y), E_y(x,y)$.

   (c) Assuming perfectly conducting walls, determine the possible values of the constants k_1, k_2 such that the E-field boundary conditions are satisfied on all three walls.

   (d) Determine the possible values of the cutoff frequency f_c of these modes. Determine the lowest cutoff frequency.



3. A load is connected to a generator by a 30-ft long 75-ohm RG-59/U coaxial cable. The SWR is measured at the load and the generator and is found to be equal to 3 and 2, respectively.

   (a) Determine the attenuation of the cable in dB/ft.

   (b) Assuming the load is resistive, what are all possible values of the load impedance in ohm?

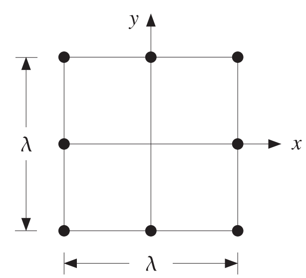
4. Eight isotropic antennas are arranged in a two-dimensional array pattern around a square of sides $\lambda\times\lambda$, as shown below, where λ is the operating wavelength.

   (a) Assuming equal array weights (i.e., unity weights) work out a real-valued expression for the array factor $A(\phi)$ as a function of the azimuthal angle ϕ, and show that it can also be expressed in the form:

$$A(\phi) = \big(2\cos(\pi\cos\phi)+1\big)\big(2\cos(\pi\sin\phi)+1\big) - 1$$

   (b) Make a rough polar plot of the array gain factor $g(\phi) = |A(\phi)|^2$ versus angle $0 \le \phi \le 360^o$ (you may use dB or absolute scales.)

   (c) How would you choose the eight array weights if the desired array pattern is to be endfire along the x-direction? Along the y-direction? Along the 45^o direction? Provide a one-sentence justification for each choice.



332:580 – Electric Waves and Radiation
Exam 1 – October 11, 2000

1. Using the BAC-CAB rule, prove the vector identity:

$$\mathbf{A}\times(\mathbf{B}\times\mathbf{C})+\mathbf{B}\times(\mathbf{C}\times\mathbf{A})+\mathbf{C}\times(\mathbf{A}\times\mathbf{B})=0$$

2. A uniform plane wave, propagating in the z-direction in vacuum, has the following electric field (where $A>0$):

$$\boldsymbol{\mathcal{E}}(t,z)=A\,\hat{\mathbf{x}}\,\cos(\omega t-kz)+2A\,\hat{\mathbf{y}}\,\sin(\omega t-kz)$$

   (a) Determine the vector phasor representing $\boldsymbol{\mathcal{E}}(t,z)$ in the complex form $\mathbf{E}=\mathbf{E}_0 e^{j\omega t-jkz}$.

   (b) Determine the polarization of this electric field (linear, circular, elliptic, left-handed, right-handed?)

   (c) Determine the magnetic field $\boldsymbol{\mathcal{H}}(t,z)$ in its real-valued form.

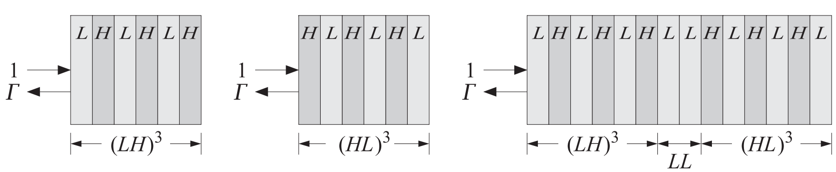
3. We wish to shield a piece of equipment from RF interference over the frequency range 100 MHZ to 1 GHz. To this end, we put the equipment in a box wrapped in aluminum foil.

   How many sheets of aluminum foil should we wrap the box in, if it is required that the external fields be attenuated by at least 70 dB inside the box?

   Assume that each aluminum foil sheet has thickness of 25.4 μm (one thousandth of an inch) and conductivity $\sigma=3.5\times10^7$ S/m.

4. The figure below shows three multilayer structures. The first, denoted by $(LH)^3$, consists of three identical bilayers, each bilayer consisting of a low-index and a high-index quarter-wave layer, with indices $n_L=1.38$ and $n_H=3.45$. The second multilayer, denoted by $(HL)^3$, is the same as the first one, but with the order of the layers reversed. The third one, denoted by $(LH)^3(LL)(HL)^3$ consists of the first two side-by-side and separated by two low-index quarter-wave layers LL.

   In all three cases, determine the overall reflection response Γ, as well as the percentage of reflected power, at the design frequency at which the individual layers are quarter-wave.



332:580 – Electric Waves and Radiation
Take-Home Midterm Exam
Due Monday, December 4, 2000

Your answer to each of the following questions must be accompanied by at least one page of text explaining your procedures. Attach any relevant graphs to each page of text. Attach your MATLAB programs as an appendix to the take-home exam.

Late exams or exams that have no explanatory text will not be accepted. Please work alone. Send me e-mail if you have any questions of clarification.

In this test, you will carry out two low-noise microwave amplifier designs, including the corresponding input and output matching networks. The first design fixes the noise figure and finds the maximum gain that can be used. The second design fixes the desired gain and finds the minimum noise figure that may be achieved.

The Hewlett-Packard Agilent ATF-34143 PHEMT transistor is suitable for low-noise amplifiers in cellular/PCS base stations, low-earth-orbit and multipoint microwave distribution systems, and other low-noise applications.

The technical data on this transistor may be found in the attached pdf file, `ATF34143.pdf`. See also the web page: `www.agilent.com/view/rf`.

At 2 GHz, its S-parameters and noise-figure data are as follows, for biasing conditions of $V_{DS}=4$ V and $I_{DS}=40$ mA (see page 9 of the pdf file):

$$S_{11}=0.700\angle{-150^{\mathrm{o}}},\qquad S_{12}=0.081\angle 19^{\mathrm{o}}$$
$$S_{21}=6.002\angle 73^{\mathrm{o}},\qquad S_{22}=0.210\angle{-150^{\mathrm{o}}}$$

$$F_{\min}=0.22\text{ dB},\quad r_n=0.09,\quad \Gamma_{G\mathrm{opt}}=0.66\angle 67^{\mathrm{o}}$$

1. At 2 GHz, the transistor is potentially unstable. Calculate the stability parameters K,μ,Δ,D_1,D_2. Calculate the MSG in dB.

   Draw a basic Smith chart and place on it the source and load stability circles (display only a small portion of each circle outside the Smith chart.)

   Then, determine the parts of the Smith chart that correspond to the source and load stability regions.

2. For the given optimum reflection coefficient $\Gamma_{G\mathrm{opt}}$, calculate the corresponding load reflection coefficient $\Gamma_{L\mathrm{opt}}$ assuming a matched load.

   Place the two points $\Gamma_{G\mathrm{opt}},\Gamma_{L\mathrm{opt}}$ on the above Smith chart and determine whether they lie in their respective stability regions.

3. Calculate the available gain $G_{a,\mathrm{opt}}$ in dB that corresponds to $\Gamma_{G\mathrm{opt}}$.

   Add the corresponding available gain circle to the above Smith chart. (Note that the source stability circle and the available gain circles intersect the Smith chart at the same points.)

4. Add to your Smith chart the noise figure circles corresponding to the noise figure values of $F = 0.25, 0.30, 0.35$ dB.

   For the case $F = 0.35$ dB, calculate and plot the available gain G_a in dB as Γ_G traces the noise-figure circle. Determine the maximum value of G_a and the corresponding value of Γ_G.

   Place on your Smith chart the available gain circle corresponding to this maximum G_a. Place also the corresponding point Γ_G, which should be the point of tangency between the gain and noise figure circles.

   Calculate and place on the Smith chart the corresponding load reflection coefficient $\Gamma_L = \Gamma_{\text{out}}^*$. Verify that the two points Γ_G, Γ_L lie in their respective stability regions.

   In addition, for comparison purposes, place on your Smith chart the available gain circles corresponding to the values $G_a = 15$ and 16 dB.

5. The points Γ_G and Γ_L determined in the previous question achieve the maximum gain for the given noise figure of $F = 0.35$ dB.

   Design input and output stub matching networks that match the amplifier to a 50-ohm generator and a 50-ohm load. Use "parallel/open" microstrip stubs having 50-ohm characteristic impedance and alumina substrate of relative permittivity of $\epsilon_r = 9.8$.

   Determine the stub lengths d, l in units of λ, the wavelength inside the microstrip lines. Choose always the solution with the shortest total length $d + l$.

   Determine the effective permittivity ϵ_{eff} of the stubs, the stub wavelength λ in cm, and the width/height ratio, w/h. Then, determine the stub lengths d, l in cm.

   Finally, make a schematic of your final design that shows both the input and output matching networks (as in Fig.10.8.3.)

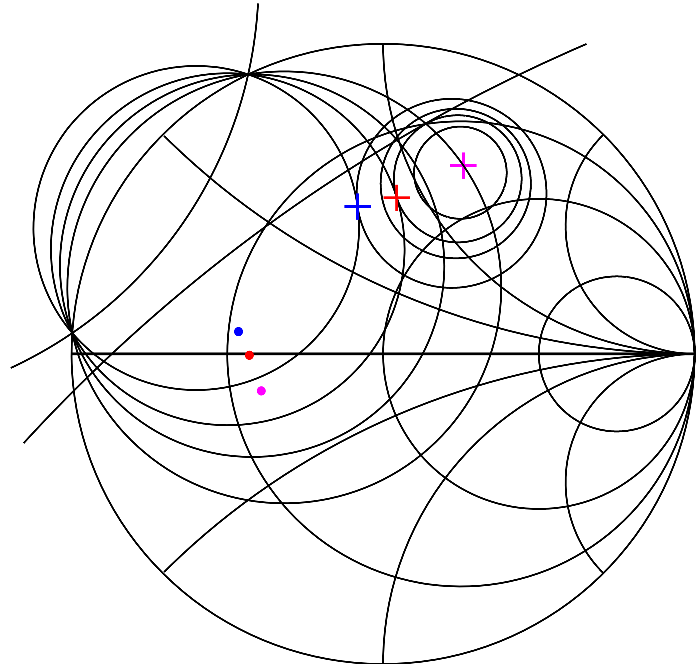
6. The above design sets $F = 0.35$ dB and finds the maximum achievable gain. Carry out an alternative design as follows. Start with a desired available gain of $G_a = 16$ dB and draw the corresponding available gain circle on your Smith chart.

   As Γ_G traces the portion of this circle that lies inside the Smith chart, compute the corresponding noise figure F. (Points on the circle can be parametrized by $\Gamma_G = c + re^{j\phi}$, but you must keep only those that have $|\Gamma_G| < 1$.)

   Find the minimum among these values of F in dB and calculate the corresponding value of Γ_G. Calculate the corresponding matched Γ_L.

   Add to your Smith chart the corresponding noise figure circle and place on it the points Γ_G and Γ_L.

7. Design the appropriate stub matching networks as in part 5.



332:580 – Electric Waves and Radiation
Final Exam – December 20, 2000

1. In an earth-satellite-earth communication system, the uplink/downlink distances are 36000 km. The uplink/downlink frequencies are 6/4 GHz. The diameters of the earth and satellite antennas are 20 m and 1 m with 60% aperture efficiencies. The transmitting earth antenna transmits power of 1.5 kW. The satellite transponder gain is 85 dB. The satellite receiving antenna is looking down at an earth temperature of 290°K and has a noisy receiver of effective noise temperature of 3000°K, whereas the earth receiving antenna is looking up at a sky temperature of 60°K and uses a high-gain LNA amplifier of noise temperature of 100°K (feedline losses may be ignored.) The bandwidth is 30 MHz.

   (a) Calculate all antenna gains in dB.

   (b) Calculate the uplink and downlink free-space losses in dB.

   (c) Calculate the amount of power received by the satellite in dBW. Calculate the uplink signal to noise ratio in dB.

   (d) Calculate the power received by the receiving earth antenna in dBW and the downlink signal to noise ratio.

   (e) Finally, calculate the total system signal to noise ratio in dB.

2. (a) Three identical isotropic antennas are placed at the corners of an equilateral triangle whose base and height have lengths equal to $\lambda/2$. The triangle lies on the xy-plane. Assuming unity array weights, determine the array factor $A(\theta,\phi)$.

   Next, take $\theta = 90°$, so that the array factor depends only on ϕ. How would you choose the array weights if you want the radiation to be directed broadside to the base?

   (b) Determine the geometry (weights, locations, sides, etc.) of the array of four isotropic antennas that has the following array factor:

$$A(\theta,\phi) = 4\cos(\pi\sin\theta\sin\phi)\cos\left(\frac{\pi}{2}\sin\theta\cos\phi\right)$$

   [*Hint:* Use Euler's formula and note that $k_x\lambda = 2\pi\sin\theta\cos\phi$.]

3. A load is connected to a generator by a 20-meter long 50-ohm coaxial cable. The SWR is measured at the load and the generator and is found to be equal to 3 and 2, respectively.

   (a) Determine the attenuation of the cable in dB/m.

   (b) Assuming that the load is resistive, what are all possible values of the load impedance in ohm? [*Hint:* the load impedance can be greater or less than the cable impedance.]

4. The electric field of the TE_{10} mode in a rectangular conducting waveguide of sides a and b is given by:

$$H_z(x) = H_0\cos k_c x\,,\quad H_x(x) = H_1\sin k_c x\,,\quad E_y(x) = E_0\sin k_c x$$

   where $b < a$, $k_c = \pi/a$, and the usual $e^{j\omega t}$ time-dependence is assumed.

   (a) Inserting these expressions into Maxwell's equations derive the relationships among the constants E_0, H_0, H_1.

   (b) By integrating the Poynting vector over the cross-sectional area of the guide, show that the total transmitted power is given by:

$$P_T = \frac{1}{4\eta}|E_0|^2 ab\sqrt{1-\frac{\omega_c^2}{\omega^2}}$$

5. A radar with EIRP of $P_{\text{radar}} = P_T G_T$ is trying to detect an aircraft of radar cross section σ. The aircraft is at a distance r from the radar and tries to conceal itself by jamming the radar with an on-board jamming antenna of EIRP of $P_{\text{jammer}} = P_J G_J$. Assume that both the radar and the jamming antennas are pointing in their direction of maximal gains.

   (a) Derive an expression of the signal-to-jammer ratio S/J, where S represents the power received from the target back at the radar antenna according to the radar equation, and J represents the power from the jamming antenna received by the radar antenna. Express the ratio in terms of P_{radar}, P_{jammer}, r, and σ.

   (b) If detectability of the target in the presence of jamming requires at least a 0-dB signal-to-jammer ratio (that is, $S/J \geq 1$), show that the maximum detectable distance is given by:

$$r = \sqrt{\frac{P_{\text{radar}}}{P_{\text{jammer}}}\frac{\sigma}{4\pi}}$$

332:580 – Electric Waves and Radiation
Exam 1 — October 9, 2002

1. (a) Consider a forward-moving wave in its real-valued form:

$$\boldsymbol{\mathcal{E}}(t,z)=\hat{\mathbf{x}}\,A\cos(\omega t-kz+\phi_a)+\hat{\mathbf{y}}\,B\cos(\omega t-kz+\phi_b)$$

Show that:

$$\boldsymbol{\mathcal{E}}(t+\Delta t,z+\Delta z)\times\boldsymbol{\mathcal{E}}(t,z)=\hat{\mathbf{z}}\,AB\sin(\phi_a-\phi_b)\sin(\omega\Delta t-k\Delta z)$$

(b) Determine the complex-phasor form of the following two real-valued fields:

$$\boldsymbol{\mathcal{E}}(t,z)\;=\;2\,\hat{\mathbf{x}}\cos(\omega t-kz)+3\,\hat{\mathbf{y}}\sin(\omega t-kz)$$

$$\boldsymbol{\mathcal{E}}(t,z)\;=\;2\,\hat{\mathbf{x}}\sin(\omega t+kz)+3\,\hat{\mathbf{y}}\cos(\omega t+kz)$$

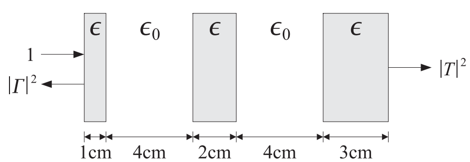
(c) Determine the propagation direction, sense of rotation, and polarization type of both of the above fields.

2. Three dielectric slabs of thicknesses of 1 cm, 2 cm, and 3 cm, and dielectric constant $\epsilon = 4\epsilon_0$ are positioned as shown below. A uniform plane wave of free-space wavelength of 8 cm is incident normally onto the left slab.

(a) Determine the power reflection and transmission coefficients, $|\Gamma|^2$ and $|T|^2$, as percentages of the incident power.

(b) Determine $|\Gamma|^2$ and $|T|^2$ if the middle slab is replaced by air.

(c) Give a one-line proof of the property $|T|^2 = 1-|\Gamma|^2$.



3. As shown below, light must be launched from air into an optical fiber at an angle $\theta \le \theta_a$ in order to propagate by total internal reflection.

(a) Show that the acceptance angle is given by:

$$\sin\theta_a=\frac{\sqrt{n_f^2-n_c^2}}{n_a}$$

(b) For a fiber of length l, show that the exiting ray, at the opposite end, is exiting at the same angle θ as the incidence angle.

(c) Show that the propagation delay time through this fiber, for a ray entering at an angle θ, is given by:

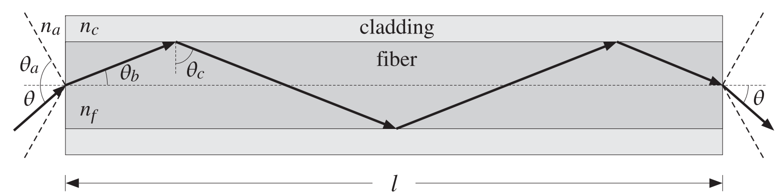
$$t(\theta)=\frac{t_0 n_f^2}{\sqrt{n_f^2-n_a^2\sin^2\theta}}$$

where $t_0 = l/c_0$.

(d) What angles θ correspond to the maximum and minimum delay times? Show that the difference between the maximum and minimum delay times is given by:

$$\Delta t = t_{\max}-t_{\min}=\frac{t_0 n_f(n_f-n_c)}{n_c}$$

Such travel time delays cause "modal dispersion," that can limit the rate at which digital data may be transmitted (typically, the data rate must be $f_{\text{bps}} \le 1/(2\Delta t)$).



332:580 – Electric Waves and Radiation
Final Exam — December 23, 2002

1. It is desired to design an air-filled rectangular waveguide such that (a) it operates only in the TE_{10} mode with the widest possible bandwidth, (b) it can transmit the maximum possible power, and (c) the operating frequency is 12 GHz and it lies in the middle of the operating band.

   (a) What are the dimensions of the guide in cm?

   (b) Taking the maximum allowed electric field to be 1 MV/m, that is, one-third the dielectric strength of air, calculate the maximum power that can be transmitted by this guide in MW.

2. A resonant dipole antenna operating in the 30-meter band is connected to a transmitter by a 30-meter long lossless coaxial cable having velocity factor of 0.8 and characteristic impedance of 50 ohm. The wave impedance at the transmitter end of the cable is measured to be 40 ohm. Determine the input impedance of the antenna.

3. The array factor of a two-element array is given by:

$$g(\phi) = |a_0 + a_1 e^{j\psi}|^2 = \frac{1+\sin\psi}{2}, \quad \psi = \frac{\pi}{2}\cos\phi$$

   where ϕ is the azimuthal angle (assume $\theta = 90^{\rm o}$) and ψ, the digital wavenumber. The array elements are along the x-axis at locations $x_0 = 0$ and $x_1 = d$.

   (a) What is the spacing d in units of λ? Determine the values of the array weights, $\mathbf{a} = [a_0, a_1]$, assuming that a_0 is real-valued and positive.

   (b) Determine the visible region and display it on the unit circle. Plot $|A(\psi)|^2$ versus ψ over the visible region. Based on this plot, make a rough sketch of the radiation pattern of the array (i.e., the polar plot of $g(\phi)$ versus $0 \le \phi \le 2\pi$).

   (c) Determine the exact 3-dB width of this array in angle space.

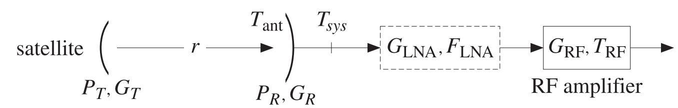
4. We showed in class that the directivity of a planar aperture antenna (with Huygens source fields) is given as follows in terms of the aperture tangential electric field:

$$D_{\max} = \frac{4\pi A_{\rm eff}}{\lambda^2} = \frac{4\pi}{\lambda^2}\,\frac{\left|\int_A \mathbf{E}_a(x,y)\,dx\,dy\right|^2}{\int_A |\mathbf{E}_a(x,y)|^2\,dx\,dy}$$

   Evaluate this expression for the case of an open-ended waveguide of sides a, b, whose aperture field is given by the TE_{10} mode. Evaluate also the

corresponding aperture efficiency $e_a = A_{\rm eff}/A_{\rm phys}$, where $A_{\rm phys}$ is the physical area of the aperture.

5. A satellite to earth downlink (shown below) is operating at the carrier frequency of 4 GHz. The distance between the two antennas is $r = 36000$ km. The bit rate is 10 Mb/s with bit error probability of $P_e = 10^{-4}$ using QPSK modulation. The satellite has transmitter power of 20 W and uses a dish antenna that has a diameter of 0.5 m and aperture efficiency of 0.6. The earth antenna has efficiency of 0.6 and antenna noise temperature of 50 K. The output of the antenna is connected to an RF amplifier with equivalent noise temperature of 1400 K.



   (a) Assuming that no LNA is used, calculate the quantities $T_{\rm sys}$, $N_0 = kT_{\rm sys}$, E_b/N_0, and the received power P_R in watts.
   For QPSK modulation, we have the relationship $P_e = {\rm erfc}\big(\sqrt{E_b/N_0}\big)/2$ with inverse $E_b/N_0 = [{\rm erfinv}(1-2P_e)]^2$. For the purposes of this exam, the following equation provides an excellent approximation to this inverse relationship over the range of $10^{-6} \le P_e \le 10^{-2}$:

$$\left(\frac{E_b}{N_0}\right)_{\rm dB} = 0.0498\,P^3 - 0.83\,P^2 + 5.60\,P - 3.91, \quad P = -\log_{10}(P_e)$$

   (b) Determine the gain G_R in dB and the diameter of the earth receiving antenna in meters.

   (c) To improve the performance of the system, a low-noise amplifier of gain of 40 dB and noise figure of 1.7 dB is inserted as shown. Assuming that the data and bit error rates remain the same, what would be the required gain and diameter of the receiving antenna?

   (d) With the LNA present and assuming that the earth antenna diameter remains as in part (b), what would be the new bit rate in Mb/s for the same bit error probability?

Hints: $k_{\rm dB} = -228.6$, $F = 1 + T_e/290$, $T_{12} = T_1 + T_2/G_1$.

332:580 – Electric Waves and Radiation
Exam 1 — October 15, 2003

1. Determine the polarization type (left, right, linear, etc.) and the direction of propagation of the following electric fields given in their phasor forms:

$$\text{a.}\quad \mathbf{E}(z) = \big[(1+j\sqrt{3})\,\hat{\mathbf{x}} + 2\,\hat{\mathbf{y}}\big]e^{+jkz}$$

$$\text{b.}\quad \mathbf{E}(z) = \big[(1+j)\,\hat{\mathbf{x}} - (1-j)\,\hat{\mathbf{y}}\big]e^{-jkz}$$

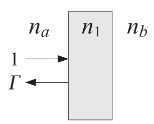
$$\text{c.}\quad \mathbf{E}(z) = \big[\hat{\mathbf{x}} - \hat{\mathbf{z}} + j\sqrt{2}\,\hat{\mathbf{y}}\big]e^{-jk(x+z)/\sqrt{2}}$$

2. A wave is normally incident from the left on a dielectric slab of refractive index n_1. The media to the left and right of the slab have indices n_a and n_b. Determine an expression for the reflection response Γ in terms of the refractive indices n_a, n_1, n_b in the following two cases:

   (a) When the operating wavelength is such that to make the slab a half-wavelength slab.

   (b) When the slab is a quarter-wavelength slab.

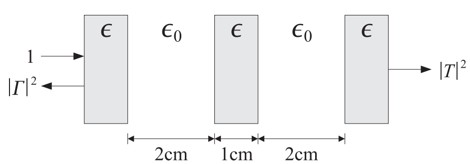
   (c) Using the expression for Γ in part (a), explain why a half-wave slab is sometimes referred to as an "absentee" layer.



3. Three identical dielectric slabs of thickness of 1 cm and dielectric constant $\epsilon_0 = 4\epsilon_0$ are positioned as shown below. A uniform plane wave of free-space wavelength of 8 cm is incident normally on the left slab.

   (a) Determine the reflectance $|\Gamma|^2$ and transmittance $|T|^2$ as percentages of the incident power.

   (b) Repeat part (a) if the two air gaps are filled with the same dielectric material resulting into a single thick slab of length of 7 cm.



4. A dielectric slab of thickness of $l = 2$ cm and refractive index $n = 2$ is immersed in air as shown below. A uniform plane wave of free-space wavelength of 8 cm is incident on the left slab at an angle θ.

   (a) Show that the transverse reflection response has the form:

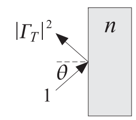
$$\Gamma_T = \frac{\rho_T(1 - e^{-2j\delta})}{1 - \rho_T^2 e^{-2j\delta}}$$

   what are ρ_T and δ as functions of the angle θ for the TE and TM polarization cases and the above specific values of l, n, λ?

   (b) At what angles do Γ_{TE} and Γ_{TM} vanish? Explain your reasoning.

   (c) Draw a sketch of the reflectances $|\Gamma_{TE}|^2$ and $|\Gamma_{TM}|^2$ versus angle θ. Indicate on your sketch the values of the reflectances at $\theta = 0^o$ and $\theta = 90^o$.

   (d) Repeat parts (b,c) if the slab thickness is reduced to $l = 1$ cm.



Hints

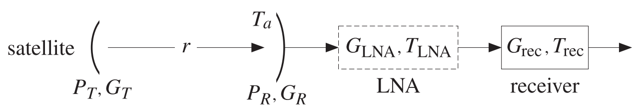
$$\boldsymbol{\mathcal{E}}(0)\times\boldsymbol{\mathcal{E}}(t) = \hat{\mathbf{z}}\,AB\cos\omega t\sin\phi$$

$$\Gamma_i = \frac{\rho_i + \Gamma_{i+1}e^{-2j\delta_i}}{1 + \rho_i\Gamma_{i+1}e^{-2j\delta_i}}\,, \quad \delta_i = 2\pi\frac{f}{f_0}L_i\cos\theta_i\,, \quad L_i = \frac{n_i l_i}{\lambda_0}$$

$$n_{TE} = n\cos\theta\,, \quad n_{TM} = \frac{n}{\cos\theta}$$

332:580 – Electric Waves and Radiation
Final Exam — December 17, 2003

1. A satellite to earth downlink (shown below) is operating at a carrier frequency of f Hertz using QPSK modulation and achieving a bit rate of R bits/sec with a bit error probability of P_e. With the LNA absent, the receiving earth antenna is connected directly to a noisy receiver with equivalent noise temperature of T_{rec}. Both antennas are dishes.



   (a) A low-noise amplifier of very high gain G_{LNA} and low noise temperature T_{LNA} is inserted between the earth antenna and the receiver. Show that the presence of the LNA allows the link to be operated (with the same error probability P_e) at the higher bit rate:

$$R_{\text{new}} = R\frac{T_a + T_{\text{rec}}}{T_a + T_{\text{LNA}}}$$

   where T_a is the earth antenna noise temperature, and $T_{\text{LNA}} \ll T_{\text{rec}}$.

   (b) The equation in part (a) is an approximation. Derive the exact form of that equation and discuss the nature of the approximation that was made.

   (c) How would the expression in part (a) change if, in addition to the assumptions of part (a), the operating frequency f were to be doubled? Explain your reasoning. How would (a) change if the transmitter power P_T were to double? If the distance r were to double?

   (d) With the LNA present, and assuming that the bit rate R, error probability P_e, and f, P_T, r remain the same, show that the diameter d of the earth antenna can be lowered to the following value without affecting performance:

$$d_{\text{new}} = d\sqrt{\frac{T_a + T_{\text{LNA}}}{T_a + T_{\text{rec}}}}$$

   where the same approximation was made as in part (a).

2. A 50-ohm lossless transmission line with velocity factor of 0.8 and operating at a frequency of 15 MHz is connected to an unknown load impedance. The voltage SWR is measured to be $S = 3 + 2\sqrt{2}$. A voltage maximum is found at a distance of 1 m from the load.

   (a) Determine the unknown load impedance Z_L.

   (b) Suppose that the line is lossy and that it is connected to the load found in part (a). Suppose that the SWR at a distance of 10 m from the load is measured to be $S = 3$. What is the attenuation of the line in dB/m?

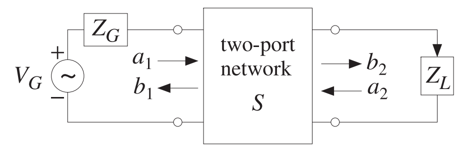
3. It is desired to design an X-band air-filled rectangular waveguide such that it operates only in the TE_{10} mode with the widest possible bandwidth, it can transmit the maximum possible power, and the operating frequency is 10 GHz and it lies in the middle of the operating band.

   (a) What are the dimensions of the guide in cm?

   (b) Taking the maximum allowed electric field to be 1.5 MV/m, that is, one-half the dielectric strength of air, calculate the maximum power that can be transmitted by this guide in kW.

   (c) Calculate the power attenuation coefficient of this guide in dB/m.

4. A two-port network with scattering matrix S is connected to a generator and load as shown below.



   (a) Show that the presence of the generator and load implies the following relationships among the input and output wave variables:

$$a_1 = \Gamma_G b_1 + b_G\,, \qquad a_2 = \Gamma_L b_2$$

   and determine b_G in terms of V_G, Z_0, Z_G (or, Γ_G).

   (b) Show that the reflection coefficient at the input of the two port is given by:

$$\Gamma_{\text{in}} = S_{11} + \frac{\Gamma_L S_{12} S_{21}}{1 - \Gamma_L S_{22}}$$

   (c) Show that the wave variables a_1, b_2 are given by:

$$a_1 = \frac{b_G}{1 - \Gamma_G \Gamma_{\text{in}}}\,, \qquad b_2 = \frac{S_{21} b_G}{(1 - \Gamma_G S_{11})(1 - \Gamma_L S_{22}) - \Gamma_G \Gamma_L S_{12} S_{21}}$$

   (d) Show that the operating power gain is given by:

$$G = \frac{P_L}{P_{\text{in}}} = \frac{\frac{1}{2}\left(|b_2|^2 - |a_2|^2\right)}{\frac{1}{2}\left(|a_1|^2 - |b_1|^2\right)} = \frac{1}{1 - |\Gamma_{\text{in}}|^2}|S_{21}|^2 \frac{1 - |\Gamma_L|^2}{|1 - S_{22}\Gamma_L|^2}$$

332:580 – Electric Waves and Radiation
Exam 1 — October 13, 2004

1. A uniform plane wave propagating in the z-direction has the following real-valued electric field:

$$\boldsymbol{\mathcal{E}}(t,z) = \hat{\mathbf{x}}\cos(\omega t - kz - \pi/4) + \hat{\mathbf{y}}\cos(\omega t - kz + \pi/4)$$

   (a) Determine the complex-phasor form of this electric field.

   (b) Determine the corresponding magnetic field $\boldsymbol{\mathcal{H}}(t,z)$ given in its real-valued form.

   (c) Determine the polarization type (left, right, linear, etc.) of this wave.

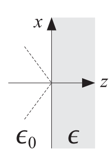
2. A single-frequency plane wave is incident obliquely from air onto a planar interface with a medium of permittivity $\epsilon = 2\epsilon_0$, as shown below. The incident wave has the following phasor form:

$$\mathbf{E}(z) = \left(\frac{\hat{\mathbf{x}} + \hat{\mathbf{z}}}{\sqrt{2}} + j\,\hat{\mathbf{y}}\right) e^{-jk(z-x)/\sqrt{2}} \qquad (1)$$

   (a) Determine the angle of incidence θ in degrees and decide which of the two dashed lines in the figure represents the incident wave. Moreover, determine the angle of refraction θ' in degrees and indicate the refracted wave's direction on the figure below.

   (b) Write an expression for the reflected wave that is similar to Eq. (1), but also includes the dependence on the TE and TM Fresnel reflection coefficients (please evaluate these coefficients numerically.) Similarly, give an expression for the transmitted wave.

   (c) Determine the polarization type (circular, elliptic, left, right, linear, etc.) of the incident wave and of the reflected wave.



3. A uniform plane wave is incident normally on a planar interface, as shown below. The medium to the left of the interface is air, and the medium to the right is lossy with an effective complex permittivity ϵ_c, complex wavenumber $k' = \beta' - j\alpha' = \omega\sqrt{\mu_0\epsilon_c}$, and complex characteristic impedance $\eta_c = \sqrt{\mu_0/\epsilon_c}$. The electric field to the left and right of the interface has the following form:

$$E_x = \begin{cases} E_0 e^{-jkz} + \rho E_0 e^{jkz}, & z \le 0 \\ \tau E_0 e^{-jk'z}, & z \ge 0 \end{cases}$$

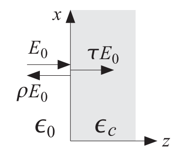
   where ρ, τ are the reflection and transmission coefficients.

   (a) Determine the magnetic field at both sides of the interface.

   (b) Show that the Poynting vector only has a z-component, given as follows at the two sides of the interface:

$$\mathcal{P} = \frac{|E_0|^2}{2\eta_0}\left(1 - |\rho|^2\right), \qquad \mathcal{P}' = \frac{|E_0|^2}{2\omega\mu_0}\beta'|\tau|^2 e^{-2\alpha' z}$$

   (c) Moreover, show that $\mathcal{P} = \mathcal{P}'$ at the interface, (i.e., at $z = 0$).

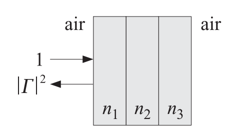


4. A radome protecting a microwave transmitter consists of a three-slab structure as shown below. The medium to the left and right of the structure is air. At the carrier frequency of the transmitter, the structure is required to be reflectionless, that is, $\Gamma = 0$.

   (a) Assuming that all three slabs are quarter-wavelength at the design frequency, what should be the relationship among the three refractive indices n_1, n_2, n_3 in order to achieve a reflectionless structure?

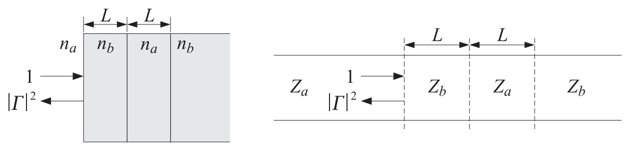
   (b) What should be the relationship among the refractive indices n_1, n_2, n_3 if the middle slab (i.e, n_2) is half-wavelength but the other two are still quarter-wavelength slabs?

   (c) For case (a), suppose that the medium to the right has a slightly different refractive index from that of air, say, $n_b = 1 + \epsilon$. Calculate the small resulting reflection response Γ to first order in ϵ.



332:580 – Electric Waves and Radiation
Final Exam — December 22, 2004

1. In order to obtain a reflectionless interface between media n_a and n_b, two dielectric slabs of equal optical lengths L and refractive indices n_b, n_a are positioned as shown below. (The same technique can be used to connect two transmission lines of impedances Z_a and Z_b.)



A plane wave of frequency f is incident normally from medium n_a. Let f_0 be the frequency at which the structure must be reflectionless. Let L be the common optical length normalized to the free-space wavelength $\lambda_0 = c_0/f_0$, that is, $L = n_a l_a/\lambda_0 = n_b l_b/\lambda_0$.

(a) Show that the reflection response into medium n_a is given by:

$$\Gamma = \rho\frac{1-(1+\rho^2)e^{-2j\delta}+e^{-4j\delta}}{1-2\rho^2 e^{-2j\delta}+\rho^2 e^{-4j\delta}}\,,\quad \rho = \frac{n_a-n_b}{n_a+n_b}\,,\quad \delta = 2\pi L\frac{f}{f_0}$$

(b) Show that the interface will be reflectionless at frequency f_0 provided the optical lengths are chosen according to:

$$L = \frac{1}{4\pi}\arccos\left(\frac{1+\rho^2}{2}\right)$$

This is known as a *twelfth-wave* transformer because for $\rho = 0$, it gives $L = 1/12$.

2. A lossless 50-ohm transmission line is connected to an unknown load impedance. Voltage measurements along the line reveal that the maximum and minimum voltage values are 6 V and 2 V. Moreover, the closest distance to the load at which a voltage minimum is observed has been found to be such that: $e^{2j\beta l_{\min}} = 0.6 - 0.6j$.

(a) Determine the load reflection coefficient Γ_L and the impedance Z_L.

(b) For a general lossless transmission line of characteristic impedance Z_0 connected to a load with a voltage standing wave ratio of S, show that the wave impedances at the locations along the line at which we observe voltage maxima or minima are given by:

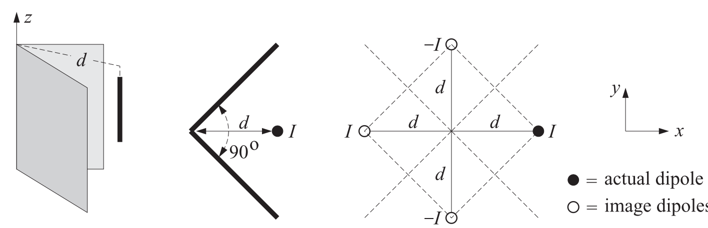
$$Z_{\max} = S\,Z_0\,,\quad Z_{\min} = \frac{1}{S}\,Z_0$$

3. The Voyager spacecraft is currently transmitting data to earth from a distance of 12 billion km. Its antenna diameter and aperture efficiency are 3.66 m and 60%. The operating frequency is 8.415 GHz and Voyager's transmitter power is 18 W. Assume the same aperture efficiency for the 70-m receiving antenna at NASA's deep-space network at Goldstone, CA.

(a) Calculate the spacecraft's and earth's antenna gains in dB. Calculate also the free-space loss in dB.

(b) Calculate the achievable communication data rate in bits/sec between Voyager and earth using QPSK modulation and assuming the following: an overall transmission loss factor of 5 dB, a system noise temperature of 25 K, an energy-per-bit to noise-spectral-density ratio of $E_b/N_0 = 3.317 = 5.208$ dB, which for QPSK corresponds to a bit-error probability of $P_e = 5\times10^{-3}$.

4. A z-directed half-wave dipole is positioned in front of a 90° corner reflector at a distance d from the corner, as shown below. The reflecting conducting sheets can be removed and replaced with three image dipoles as shown. The image dipoles lying along the y-direction have opposite currents compared to the original dipole, whereas the dipole along x has the same phase.



(a) Thinking of the equivalent image problem as an array, determine its array factor $A(\phi)$ as a function of the azimuthal angle ϕ on the xy-plane.

(b) For the case $d = \lambda/2$, make a rough sketch of $|A(\phi)|^2$ versus ϕ in the range $0 \le \phi \le 45°$. The 3-dB angle ϕ_3 turns out to be close to 21°. Determine a more precise value for this angle.

332:580 – Electric Waves and Radiation
Exam 1 — October 12, 2005

1. Determine the polarization type (left, right, linear, etc.) and the direction of propagation of the following electric fields given in their real-valued forms:

   a. $\boldsymbol{\mathcal{E}}(t,z) = \hat{\mathbf{x}}\cos(\omega t - kz) + \hat{\mathbf{y}}\sin(\omega t - kz)$
   b. $\boldsymbol{\mathcal{E}}(t,z) = \hat{\mathbf{x}}\cos(\omega t - kz) - \hat{\mathbf{y}}\sin(\omega t - kz)$
   c. $\boldsymbol{\mathcal{E}}(t,z) = \hat{\mathbf{x}}\sin(\omega t - kz) + \hat{\mathbf{y}}\cos(\omega t - kz)$
   d. $\boldsymbol{\mathcal{E}}(t,z) = \hat{\mathbf{x}}\sin(\omega t - kz) - \hat{\mathbf{y}}\cos(\omega t - kz)$

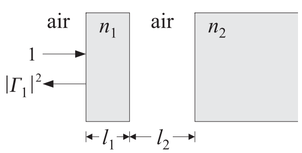
2. Consider the following linearly-polarized wave given in its real-valued form, where A, B are positive amplitudes:

$$\boldsymbol{\mathcal{E}}(t,z) = A\hat{\mathbf{x}}\cos(\omega t - kz) + B\hat{\mathbf{y}}\cos(\omega t - kz)$$

   Show that it can be expressed as a linear combination of two circularly polarized waves, one left- and the other right-circular. Express these circularly-polarized waves in their real-valued forms. Moreover, show that the right (resp. left) polarized wave can itself be written as the linear combination of two separate right (resp. left) polarized waves, one having amplitude $A/2$ and the other amplitude $B/2$.

3. A lossless dielectric slab of refractive index n_1 and thickness l_1 is positioned at a distance l_2 from a semi-infinite dielectric of refractive index n_2, as shown below.

   A uniform plane wave of free-space wavelength λ_0 is incident normally on the slab from the left. Assuming that the slab n_1 is a quarter-wavelength slab, determine the length l_2 (in units of λ_0) and the relationship between n_1 and n_2 in order that there be no reflected wave into the leftmost medium (i.e., $\Gamma_1 = 0$).

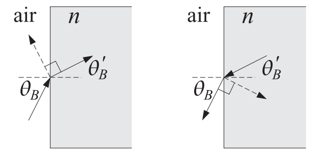


4. A TM polarized wave is incident from air onto a planar dielectric interface at the Brewster angle θ_B, as shown below. Let θ'_B be the refracted angle. The right medium is lossless and has refractive index n.

   (a) Derive expressions for θ_B and θ'_B in terms of n.

   (b) Show that $\theta_B + \theta'_B = 90^{\circ}$ and that the angle between the refracted ray and the would-be reflected ray is 90°.

   (c) By reversing the direction of the refracted ray, show that θ'_B is the Brewster angle for a wave going from the medium n into the air (in the sense that there would be no TM reflected wave from the right of the interface.)



332:580 – Electric Waves and Radiation
Exam 2 — December 5, 2005

1. Consider a coaxial transmission line of inner and outer radii a and b. The inner conductor is held at voltage V, and has a charge Q' per unit length, and a current I. The outer conductor is grounded (with charge $-Q'$ and current $-I$.)

   (a) Using Ampére's and Gauss' laws, show that the magnetic and electric fields at a distance $a \le r \le b$ are given by:

   $$H_\phi = \frac{I}{2\pi r}\,, \quad E_r = \frac{Q'}{2\pi\epsilon r}$$

   (b) Using these results, determine expressions for the inductance and capacitance per unit length L', C'.

2. A lossless 50-ohm transmission line is connected to an unknown load impedance Z_L. Voltage measurements along the line reveal that the maximum and minimum voltage values are $(\sqrt{2}+1)$ volts and $(\sqrt{2}-1)$ volts. Moreover, a distance at which a voltage maximum is observed has been found to be $l_{\max} = 15\lambda/16$.

   (a) Determine the load reflection coefficient Γ_L and the impedance Z_L.

   (b) Determine a distance (in units of λ) at which a voltage minimum will be observed.

   (c) Let Z_l be the wave impedance at a distance l from the load. Show that it is bounded from above and below as follows, where Z_0 is the characteristic impedance of the line, and S, the standing wave ratio:

   $$\frac{1}{S}Z_0 \le |Z_l| \le SZ_0$$

   At what lengths l are the limits realized?
   Hint: $\big||a| - |b|\big| \le |a+b| \le |a| + |b|$.

3. A 50-ohm transmission line is terminated at a load impedance:

   $$Z_L = 75 + j25\ \Omega$$

   (a) What percentage of the incident power is reflected back into the line?

   (b) In order to make the load reflectionless, a short-circuited 50-ohm stub of length d is inserted in parallel at a distance l from the load. What are the smallest values of the lengths d and l in units of the wavelength λ that will make the load reflectionless? Show all work.

332:580 – Electric Waves and Radiation
Final Exam — December 21, 2005

1. An open-ended waveguide operating in its TE_{10} mode is radiating into free space from its open end.

   Explain why the directivity of the radiating waveguide aperture is given as follows, where $E_y(x')$ is the aperture electric field and the waveguide sides are $a > b$:

$$D_{\max} = \frac{4\pi A_{\text{eff}}}{\lambda^2} = \frac{4\pi b}{\lambda^2}\,\frac{\left|\displaystyle\int_{-a/2}^{a/2} E_y(x')\,dx'\right|^2}{\displaystyle\int_{-a/2}^{a/2} |E_y(x')|^2\,dx'}$$

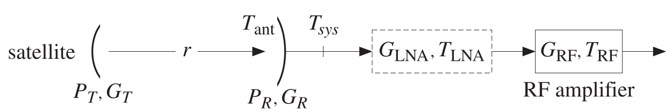
   By evaluating this expression, show that the aperture efficiency of this waveguide is:

$$e_a = \frac{A_{\text{eff}}}{A_{\text{phys}}} = \frac{8}{\pi^2} = 0.81$$

   Hint: Make sure your expression for $E_y(x')$ satisfies the boundary conditions at the waveguide walls.

2. A satellite to earth downlink (shown below) is operating at the carrier frequency of 4 GHz. The distance between the two antennas is $r = 40\,000$ km. The bit error probability is $P_e = 10^{-5}$ using QPSK modulation. For QPSK and this error probability, the quantity E_b/N_0 is equal to 9.0946 (in absolute units).

   The satellite has transmitter power of 20 W and uses a dish antenna that has a diameter of 0.5 m and aperture efficiency of 0.6. The earth antenna has diameter of 5 m, efficiency of 0.6, and antenna noise temperature of 50 K. The output of the antenna is connected to an RF amplifier with equivalent noise temperature of 2000 K.



   (a) Assuming that no LNA is used, calculate the system noise temperature T_{sys} at the output of the receiving antenna, the received power P_R in picowatts, and the maximum achievable data rate in Mb/sec.

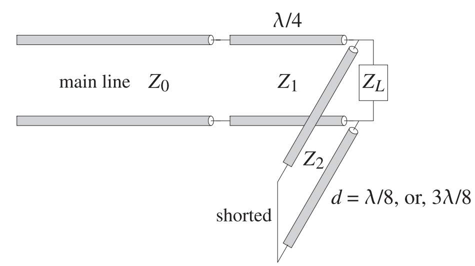
   (b) It is desired to improved the performance of this system tenfold, that is, to increase the maximum achievable data rate in Mb/sec by a factor of 10. To this end, a low-noise amplifier of 40-dB gain is inserted as shown. Determine the noise temperature of the LNA that would guarantee such a performance improvement.

   (c) What is the maximum noise temperature of the LNA that can achieve such a 10-fold improvement, and at what LNA gain is it achieved?

3. A 50-ohm transmission line is terminated at the load impedance:

$$Z_L = 40 + 80j\ \Omega$$

   (a) In order to make the load reflectionless, a quarter-wavelength transformer section of impedance Z_1 is inserted between the line and the load, as show below, and a $\lambda/8$ or $3\lambda/8$ short-circuited stub of impedance Z_2 is inserted in parallel with the load.



   Determine the characteristic impedances Z_1 and Z_2 and whether the parallel stub should have length $\lambda/8$ or $3\lambda/8$.

   (b) In the general case of a shorted stub, show that the matching conditions are equivalent to the following relationship among the quantities Z_0, Z_L, Z_1, Z_2:

$$Z_L = \frac{Z_0 Z_1^2 Z_2^2 \pm jZ_2 Z_1^4}{Z_0^2 Z_2^2 + Z_1^4}$$

   where Z_0, Z_1, Z_2 are assumed to be lossless. Determine which $\pm$ sign corresponds to $\lambda/8$ or $3\lambda/8$ stub length.

332:580 – Electric Waves and Radiation
332:481 – Electromagnetic Waves
Exam 1 — October 11, 2006

1. Ground-penetrating radar is used to detect underground objects. Assuming that the earth has conductivity $\sigma = 10^{-3}$ S/m, permittivity $\epsilon = 9\epsilon_0$, and permeability $\mu = \mu_0$, determine the maximum depth of detecting an object if detectability requires that the roundtrip attenuation (from the surface to the object and back to the surface) is not greater than 30 dB. The radar is operating at 900 MHz. [*Hint:* the roundtrip amplitude attenuation to depth z is $e^{-2\alpha z}$.]

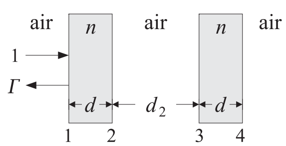
2. Consider the two electric fields, one given in its real-valued form, and the other, in its phasor form:

   a. $\boldsymbol{\mathcal{E}}(t,z) = \hat{\mathbf{x}}\sin(\omega t + kz) + 2\hat{\mathbf{y}}\cos(\omega t + kz)$
   b. $\mathbf{E}(z) = \big[(1+j)\hat{\mathbf{x}} - (1-j)\hat{\mathbf{y}}\big]e^{-jkz}$

   For both cases, determine the polarization of the wave (linear, circular, left, right, etc.) and the direction of propagation.

   For case (a), determine the field in its phasor form. For case (b), determine the field in its real-valued form as a function of t, z.

3. In order to provide structural strength and thermal insulation, a radome is constructed using two identical dielectric slabs of length d and refractive index n, separated by an air-gap of length d_2, as shown below.



   Recall that a reflectionless single-layer radome requires that the dielectric layer have half-wavelength thickness.

   However, show that for the above dual-slab arrangement, either half- or quarter-wavelength dielectric slabs may be used, provided that the middle air-gap is chosen to be a half-wavelength layer, i.e., $d_2 = \lambda_0/2$, at the operating wavelength λ_0. [*Hint:* Work with wave impedances at the operating wavelength.]

4. For the previous problem, determine an expression of the reflection response Γ at λ_0 in terms of the refractive index n for the following two choices of the air-gap length:

   (a) $d_2 = \lambda_0/4$, quarter-wavelength.

   (b) $d_2 = \lambda_0/8$, eighth-wavelength.

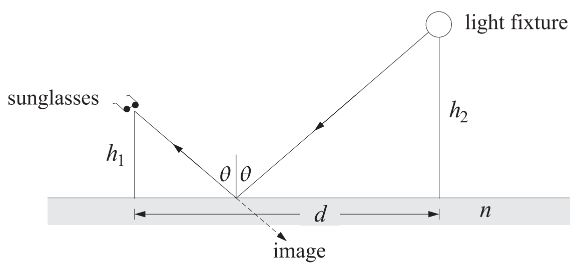
   [*Hint:* As a test for $n = 1.5$, the value is $\Gamma = -0.6701$ for case (a), and $\Gamma = -0.4321 - 0.3207j$ for case (b).]

332:580 – Electric Waves and Radiation
Exam 2 — November 15, 2006

1. You are walking along the hallway in your classroom building wearing polaroid sunglasses and looking at the reflection of a light fixture on the waxed floor. Suddenly, at a distance d from the light fixture, the reflected image momentarily disappears. Show that the refractive index of the reflecting floor can be determined from the ratio of distances:

$$n = \frac{d}{h_1 + h_2}$$

where h_1 is your height and h_2 that of the light fixture. You may assume that light from the fixture is unpolarized, that is, a mixture of 50% TE and 50% TM, and that the polaroid sunglasses are designed to filter out horizontally polarized light. Explain your reasoning.



2. Design an X-band rectangular air-filled waveguide to be operated at 10 GHz. The operating frequency must lie in the middle of the operating band. Calculate the guide dimensions in cm, the attenuation constant in dB/m, and the maximum transmitted power in kW assuming that the maximum allowable electric field is one-half of the dielectric strength of air (3 MV/m). Assume copper walls with conductivity $\sigma = 5.8\times10^7$ S/m.

3. The wavelength on a 50-ohm transmission line is 8 meters. Determine the load impedance if the SWR on the line is 3 and the location of the first voltage maximum is 1 meter from the load.

   At what other distances from the load would one measure a voltage minimum? A voltage maximum?

4. A 10-volt generator with a 25-ohm internal impedance is connected to a 100-ohm load via a 6-meter long 50-ohm transmission line. The wavelength on the line is 8 meters. Carry out the following calculations in the stated order:

   (a) Calculate the wave impedance Z_d at the generator end of the line. Then, using an equivalent voltage divider circuit, calculate the voltage and current V_d, I_d. Then, calculate the forward and backward voltages V_{d+}, V_{d-} from the knowledge of V_d, I_d.

   (b) Propagate V_{d+}, V_{d-} to the load end of the line to determine the values of the forward and backward voltages V_{L+}, V_{L-} at the load end. Then, calculate the corresponding voltage and current V_L, I_L from the knowledge of V_{L+}, V_{L-}.

   (c) Assuming that the real-valued form of the generator voltage is

$$V_G = 10\cos(\omega t)$$

   determine the real-valued forms of the quantities V_d, V_L expressed in the sinusoidal form $A\cos(\omega t + \theta)$.

Hint: $e^{j\pi} = -1$, $e^{j\pi/2} = j$.

332:580 – Electric Waves and Radiation
Final Exam — December 18, 2006

1. It is desired to design an air-filled rectangular waveguide operating at 3 GHz, whose group velocity is 0.6c.

   (a) What are the dimensions a, b of the guide (in cm) if it is also required to carry maximum power and have the widest possible bandwidth?

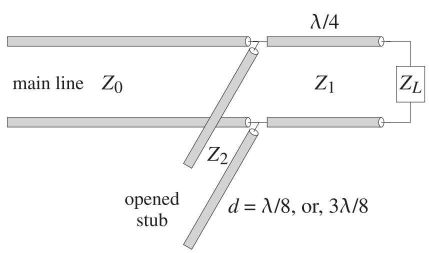
   (b) What is the cutoff frequency of the guide in GHz and the operating bandwidth?

2. The SWR on a lossy 50-ohm line is measured to be equal to 3 at distance of 5 meters from the load, and equal to 4 at a distance of 1 meter from the load.

   (a) Determine the attenuation constant of the line in dB/m.

   (b) Assuming that the load is purely resistive, determine the two possible values of the load impedance. [*Hint:* one of the two values is 11.4 ohm.]

3. It is desired to match a transmission line with characteristic impedance Z_0 to a complex load $Z_L = R_L + jX_L$. In order to make the load reflectionless, a quarter-wavelength transformer section of impedance Z_1 is inserted between the main line and the load, and a $\lambda/8$ or $3\lambda/8$ open-circuited stub of impedance Z_2 is inserted in parallel with the load, as shown below.

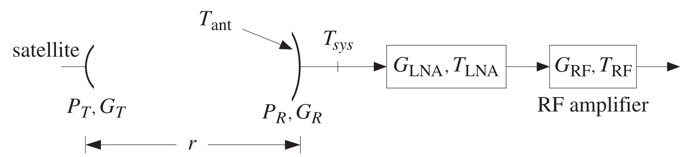


   (a) Determine expressions for Z_1 and Z_2 in terms of Z_0, R_L, X_L. Moreover, depending on the sign of X_L, decide when one should use a $\lambda/8$ or a $3\lambda/8$ stub. Please note that Z_0, Z_1, Z_2 are real and positive quantities.

   (b) Are there any impedances Z_L for which this method will not work? What would be a simple modification of this method (i.e., still using one or both of the $\lambda/4$ and $\lambda/8$ segments) that should be applied in such cases?

4. A satellite to earth link (shown below) is operating at the carrier frequency of 4 GHz. The data link employs QPSK modulation and achieves a bit-error-rate probability of $P_e = 10^{-6}$. The satellite has transmitter power of 20 W and uses a dish antenna that has a diameter of 0.5 m and aperture efficiency of 0.6. The earth antenna has diameter of 2 m, efficiency of 0.6, and antenna noise temperature of 80 K. The satellite antenna is at a distance of 40,000 km from the earth antenna.

   The output of the receiving antenna is connected to a high-gain low noise amplifier with gain of 40 dB and equivalent noise temperature of 200 K. The output of the LNA is connected to an RF amplifier with equivalent noise temperature of 1800 K.



   For QPSK modulation, we have the relationship $P_e = \text{erfc}\big(\sqrt{E_b/N_0}\big)/2$ with inverse $E_b/N_0 = [\text{erfinv}(1-2P_e)]^2$. For the purposes of this exam, the following equation provides an excellent approximation to this inverse relationship over the range of $10^{-8} \le P_e \le 10^{-3}$:

$$\frac{E_b}{N_0} = -2.1969\,\log_{10}(P_e) - 1.8621$$

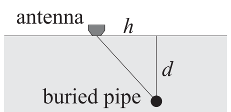
   where E_b/N_0 is in absolute units.

   (a) Calculate the achievable communication data rate R in megabits/sec.

   (b) If the LNA is removed, the performance of the system will deteriorate. In an attempt to keep the data rate the same as in part (a), the satellite transmitter power is increased to 80 W. Calculate the deteriorated value of the bit-error-rate P_e in this case.

332:580 – Electric Waves and Radiation
332:481 – Electromagnetic Waves
Exam 1 — October 10, 2007

1. Ground-penetrating radar operating at 900 MHz is used to detect underground objects, as shown in the figure below for a buried pipe. Assume that the earth has conductivity $\sigma = 10^{-3}$ S/m, permittivity $\epsilon = 9\epsilon_0$, and permeability $\mu = \mu_0$. You may use the "weakly lossy dielectric" approximation.



(a) Determine the numerical value of the wavenumber $k = \beta - j\alpha$ in meters^{-1}, and the penetration depth $\delta = 1/\alpha$ in meters.

(b) Determine the value of the complex refractive index $n_c = n_r - jn_i$ of the ground at 900 MHz.

(c) With reference to the above figure, explain why the electric field returning back to the radar antenna after getting reflected by the buried pipe is given by

$$\left|\frac{E_{\text{ret}}}{E_0}\right|^2 = \exp\left[-\frac{4\sqrt{h^2+d^2}}{\delta}\right]$$

where E_0 is the transmitted signal, d is the depth of the pipe, and h is the horizontal displacement of the antenna from the pipe. You may ignore the angular response of the radar antenna and assume it emits isotropically in all directions into the ground.

(d) The depth d may be determined by measuring the roundtrip time $t(h)$ of the transmitted signal at successive horizontal distances h. Show that $t(h)$ is given by:

$$t(h) = \frac{2n_r}{c_0}\sqrt{d^2+h^2}$$

where n_r is the real part of the complex refractive index n_c.

(e) Suppose $t(h)$ is measured over the range $-2 \le h \le 2$ meters over the pipe and its minimum recorded value is $t_{\text{min}} = 0.2$ μsec. What is the depth d in meters?

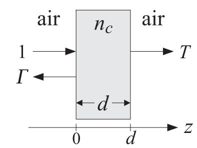
2. A uniform plane wave propagating in vacuum along the z direction has real-valued electric field components:

$$\mathcal{E}_x(z,t) = \cos(\omega t - kz)\,, \quad \mathcal{E}_y(z,t) = 2\sin(\omega t - kz)$$

(a) Its phasor form has the form $\mathbf{E} = (A\,\hat{\mathbf{x}} + B\,\hat{\mathbf{y}})e^{\pm jkz}$. Determine the numerical values of the complex-valued coefficients A, B and the correct sign of the exponent.

(b) Determine the polarization of this wave (left, right, linear, etc.). Explain your reasoning.

3. Consider a lossy dielectric slab of thickness d and complex refractive index $n_c = n_r - jn_i$ at an operating frequency ω, with air on both sides as shown below.



(a) Let $k = \beta - j\alpha = k_0 n_c$ and $\eta_c = \eta_0/n_c$ be the corresponding complex wavenumber and characteristic impedance of the slab, where $k_0 = \omega\sqrt{\mu_0\epsilon_0} = \omega/c_0$ and $\eta_0 = \sqrt{\mu_0/\epsilon_0}$. Show that the transmission response of the slab may be expressed as follows:

$$T = \frac{1}{\cos kd + j\dfrac{1}{2}\left(n_c + \dfrac{1}{n_c}\right)\sin kd}$$

Hint: $\rho_1 = (\eta_c - \eta_0)/(\eta_c + \eta_0)$ and $\rho_2 = -\rho_1$.

(b) At the cell phone frequency of 900 MHz, the complex refractive index of concrete is $n_c = 2.5 - 0.14j$. Calculate the percentage of the transmitted power through a 20-cm concrete wall. How is this percentage related to T and why?
Hint: $\cos(kd) = -1.14$, $\sin(kd) = 0.55j$, and $1/n_c = 0.399 + 0.022j$.

(c) Is there anything interesting about the choice $d = 20$ cm? Explain.
[*Hint:* $c_0 = 30$ cm $\cdot$ GHz.]

4. Consider the slab of the previous problem. The tangential electric field has the following form in the three regions $z \le 0$, $0 \le z \le d$, and $z \ge d$:

$$E(z) = \begin{cases} e^{-jk_0 z} + \Gamma e^{jk_0 z}\,, & \text{if} \quad z \le 0 \\ Ae^{-jkz} + Be^{jkz}\,, & \text{if} \quad 0 \le z \le d \\ Te^{-jk_0(z-d)}\,, & \text{if} \quad z \ge d \end{cases}$$

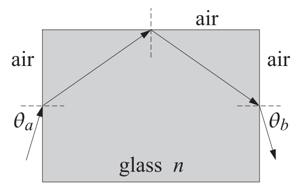
where k_0 and k were defined in the previous problem.

(a) What are the corresponding expressions for the magnetic field $H(z)$?

(b) Set up—but do not solve—four equations from which the four unknowns Γ, A, B, T may be determined.

332:580 – Electric Waves and Radiation
332:481 – Electromagnetic Waves
Exam 2 — November 14, 2007

1. A light ray enters a glass block from one side, suffers a total internal reflection from the top side, and exits from the opposite side, as shown below. The glass refractive index is $n = 1.5$.

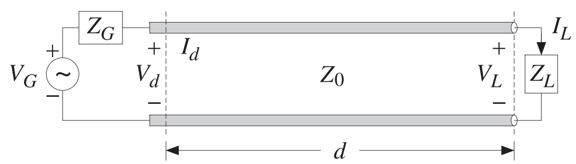


   (a) How is the exit angle θ_b related to the entry angle θ_a? Explain.

   (b) Show that all rays, regardless of the entry angle θ_a, will suffer total internal reflection at the top side.

   (c) Suppose that the glass block is replaced by another dielectric with refractive index n. What is the minimum value of n in order that all entering rays will suffer total internal reflection at the top side?

2. A lossless 50-ohm transmission line of length $d = 17$ m is connected to an unknown load Z_L and to a generator $V_G = 10$ volts having an unknown internal impedance Z_G, as shown below. The wavelength on the line is $\lambda = 8$ m. The current and voltage on the line at the generator end are measured and found to be $I_d = 40$ mA and $V_d = 6$ volts.



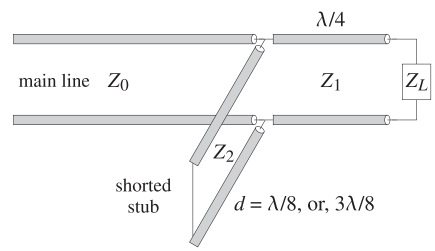
   (a) Determine the wave impedance Z_d at the generator end, as well as the generator's internal impedance Z_G.

   (b) Determine the load impedance Z_L.

   (c) What percentage of the total power produced by the generator is absorbed by the load?

3. The SWR on a lossy line is measured to be equal to 3 at a distance of 5 meters from the load, and equal to 4 at a distance of 1 meter from the load. Determine the attenuation constant of the line in dB/m.

4. It is desired to match a line with characteristic impedance Z_0 to a complex load $Z_L = R_L + jX_L$. In order to make the load reflectionless, a quarter-wavelength section of impedance Z_1 is inserted between the main line and the load, and a $\lambda/8$ or $3\lambda/8$ short-circuited stub of impedance Z_2 is inserted in parallel at the end of the line, as shown below.



   (a) Show that the section characteristic impedances must be chosen as:

$$Z_1 = \sqrt{Z_0 R_L}\,, \quad Z_2 = Z_0 \frac{R_L}{|X_L|}$$

   Such segments are easily implemented with microstrip lines.

   (b) Depending on the sign of X_L, decide when one should use a $\lambda/8$ or a $3\lambda/8$ stub.

   (c) The above scheme works if both R_L and X_L are non-zero. What should we do if $R_L \neq 0$ and $X_L = 0$? What should we do if $R_L = 0$ and $X_L \neq 0$?

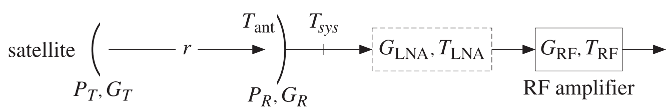
332:580 – Electric Waves and Radiation
332:481 – Electromagnetic Waves
Final Exam — December 21, 2007

1. A satellite to earth downlink (shown below) is operating at the carrier frequency of 4 GHz. The distance between the two antennas is $r = 40\,000$ km. The bit error probability is $P_e = 10^{-5}$ using QPSK modulation.

   For QPSK modulation, we have the following relationship between the bit-error-probability and E_b/N_0 ratio, expressed in terms of the MATLAB functions `erfc` and `erfinv`:

$$P_e = \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right) \quad \Leftrightarrow \quad \frac{E_b}{N_0} = \left[\mathrm{erfinv}(1-2P_e)\right]^2$$

   The satellite has transmitter power of 20 W and uses a dish antenna that has a diameter of 0.5 m and aperture efficiency of 0.6. The earth antenna has diameter of 5 m, efficiency of 0.6, and antenna noise temperature of 50 K. The output of the antenna is connected to an RF amplifier with equivalent noise temperature of 2000 K.

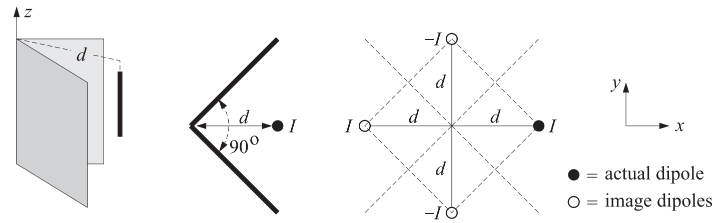


   (a) Assuming that no LNA is used, calculate the system noise temperature $T_{\rm sys}$ at the output of the receiving antenna, the received power P_R in picowatts, and the maximum achievable data rate in Mb/sec.

   (b) It is desired to improved the performance of this system tenfold, that is, to increase the maximum achievable data rate in Mb/sec by a factor of 10. To this end, a low-noise amplifier of 40-dB gain is inserted as shown. Determine the noise temperature of the LNA that would guarantee such a performance improvement.

   (c) What is the maximum noise temperature of the LNA that can achieve such a 10-fold improvement, and at what LNA gain is it achieved?

2. A z-directed half-wave dipole is positioned in front of a 90° corner reflector at a distance d from the corner, as shown below. The reflecting conducting sheets can be removed and replaced by three image dipoles of alternating signs, as shown.



   (a) Thinking of the equivalent image problem as an array, determine an analytical expression for the array factor $A(\theta,\phi)$ as a function of the polar and azimuthal angles θ,ϕ.

   (b) For the values $d = 0.5\lambda$, $d = \lambda$, and $d = 1.5\lambda$, plot the azimuthal pattern $A(90^\circ,\phi)$ at polar angle $\theta = 90^\circ$ and for $-45^\circ \le \phi \le 45^\circ$.

   (c) For the cases $d = 0.5\lambda$ and $d = 1.5\lambda$, calculate the directivity D (in dB and in absolute units) and compare it with the directivity of a single half-wave dipole in the absence of the reflector.

   (d) Suppose that the corner reflector is flattened into a conducting sheet lying on the yz plane, i.e., the 90° angle between the sheets is replaced by a 180° angle. Repeat parts (a–c) in this case.

3. For this problem you will need to read ch.21 of the text and the attached papers. A short summary is given below. The current on a thin linear antenna is determined from the solution of Hallén's integral equation, which takes the following two forms for the cases of a delta-gap excitation and for a plane wave incident on the antenna at an angle θ (see Fig. 21.2.1),

$$\int_{-h}^{h} Z(z-z')I(z')\,dz' = V(z) = C_1\cos kz + V_0\sin k|z|$$

$$\int_{-h}^{h} Z(z-z')I(z')\,dz' = V(z) = C_1 e^{jkz} + C_2 e^{-jkz} + \frac{2E_0}{k\sin\theta}e^{jkz\cos\theta}$$

   where h is the half-length, $h = l/2$, of the antenna with length l, and the other quantities are defined in Sections 21.1–21.3. The constants C_1, C_2 are determined by requiring that the current vanish at the antenna endpoints, that is, $I(h) = I(-h) = 0$.

   In this problem, you will study the properties of the numerical solution of these equations using the method of moments (MoM), and in particular, using a pulse-function basis and either point-matching or Galerkin's weighting functions. The MoM approach is as follows. The antenna is divided into $N = 2M+1$ segments of width $\Delta = l/N = 2h/(2M+1)$ with centers at the positions (see Fig. 21.7.1, type-1 case):

$$z_m = m\Delta\,, \quad -M \le m \le M$$

and the current is expanded into pulse-basis functions as in Eq. (21.8.2):

$$I(z') = \sum_{m=-M}^{M} I_m B(z' - z_m)$$

where

$$B(z' - z_m) = \begin{cases} 1, & \text{if} \quad |z' - z_m| \le \frac{1}{2}\Delta \\ 0, & \text{otherwise} \end{cases}$$

Substitution of $I(z')$ into the Hallén equation gives:

$$\sum_{m=-M}^{M} I_m \int_{-h}^{h} Z(z - z')B(z' - z_m)dz' = V(z)$$

Next, a local average is formed about each point $z = z_n = n\Delta$ by using a local weighting function $W(z - z_n)$:

$$\sum_{m=-M}^{M} I_m \int_{-h}^{h}\int_{-h}^{h} W(z-z_n)Z(z-z')B(z'-z_m)\, dz\, dz' = \int_{-h}^{h} W(z-z_n)V(z)\, dz$$

This may be written in the $N\times N$ matrix form:

$$\sum_{m=-M}^{M} Z_{nm}I_m = V_n\,, \quad -M \le n \le M$$

where

$$Z_{nm} = \int_{-h}^{h}\int_{-h}^{h} W(z - z_n)Z(z - z')B(z' - z_m)\, dz\, dz'$$

$$V_n = \int_{-h}^{h} W(z - z_n)V(z)\, dz$$

In the Galerkin method the weighting function is taken to be the same as the basis function, and in the point-matching case, it is a delta function:

$$W(z - z_n) = \delta(z - z_n) \qquad \text{(point-matching)}$$
$$W(z - z_n) = B(z - z_n) \qquad \text{(Galerkin)}$$

Thus, in the point-matching method, Z_{nm} and V_n will be:

$$Z_{nm} = \int_{-\Delta/2}^{\Delta/2} Z(z_n - z_m + x)\, dx \quad \text{and} \quad V_n = V(z_n)$$

so that V_n is given as follows in the delta-gap and plane-wave cases:

$$V_n = C_1 \cos kz_n + V_0 \sin k|z_n|$$

$$V_n = C_1 e^{jkz_n} + C_2 e^{-jkz_n} + \frac{2E_0}{k\sin\theta}\, e^{jkz_n\cos\theta}$$

with $z_n = n\Delta$, $-M \le n \le M$. Similarly, in the Galerkin case, we have:

$$Z_{nm} = \int_{-\Delta}^{\Delta} (\Delta - |x|)Z(z_n - z_m + x)\, dx$$

and V_n is given as follows in the delta-gap and plane-wave cases (where $\delta(n)$ is the Kronecker delta):

$$V_n = \frac{2}{k}\sin\frac{k\Delta}{2}\left(C_1\cos kz_n + V_0 \sin k|z_n|\right) + V_0\delta(n)\frac{4}{k}\sin^2\frac{k\Delta}{4}$$

$$V_n = \frac{2}{k}\sin\frac{k\Delta}{2}\left(C_1 e^{jkz_n} + C_2 e^{-jkz_n}\right) + \frac{2\sin\left(\dfrac{k\Delta\cos\theta}{2}\right)}{k\cos\theta}\,\frac{2E_0}{k\sin\theta}\, e^{jkz_n\cos\theta}$$

The resulting $N\times N$ matrix equation for the current can be written in the following compact forms in the delta-gap and plane-wave cases:

$$\mathcal{Z}\boldsymbol{I} = C_1\,\mathbf{c} + V_0\,\mathbf{s} \qquad \text{(delta-gap)}$$
$$\mathcal{Z}\boldsymbol{I} = C_1\,\mathbf{c}_1 + C_2\,\mathbf{c}_2 + E_0\,\mathbf{s} \qquad \text{(plane wave)}$$

with appropriate definitions for the vectors $\mathbf{c}, \mathbf{s}, \mathbf{c}_1, \mathbf{c}_2$ depending on using point-matching or the Galerkin method, where $\mathcal{Z}$ is the matrix $[Z_{nm}]$ and $\boldsymbol{I}$ is the N-dimensional vector of current samples:

$$\boldsymbol{I} = \begin{bmatrix} I_{-M} \\ \vdots \\ I_{-1} \\ I_0 \\ I_1 \\ \vdots \\ I_M \end{bmatrix}$$

Sections 21.7–21.9 discuss how to solve these equations for $\boldsymbol{I}$ and the constants C_1, C_2, subject to the end-conditions $I_{-M} = I_M = 0$. Note that in the delta-gap case, the current is symmetric about its middle, and therefore only the lower half of the vector $\boldsymbol{I}$ is needed. The text explains how to wrap the linear system in half in this case.

The matrix elements Z_{nm} can be written in the following simpler forms that use only half of the integration ranges:

$$Z_{nm} = \int_0^{\Delta/2} \big[Z(z_n - z_m + x) + Z(z_n - z_m - x)\big]dx \qquad \text{(point-match)}$$

$$Z_{nm} = \int_0^{\Delta} (\Delta - x)\big[Z(z_n - z_m + x) + Z(z_n - z_m - x)\big]dx \qquad \text{(Galerkin)}$$

These integrals can be done numerically using Gauss-Legendre quadrature integration. For example, using a J-point integration rule, we may write:

$$Z_{nm} = \sum_{j=1}^{J} \big[Z(z_n - z_m + x_j) + Z(z_n - z_m - x_j)\big]w_j$$

$$Z_{nm} = \sum_{j=1}^{J} (\Delta - x_j)\big[Z(z_n - z_m + x_j) + Z(z_n - z_m - x_j)\big]w_j$$

where w_j, x_j are the quadrature weights and evaluation points, with respect to the integration interval $[0, \Delta/2]$, or $[0, \Delta]$ in the second case, and can be obtained by calling the MATLAB function `quadr` as follows (the value $J = 32$ is recommended):

$$[\mathbf{w}, \mathbf{x}] = \text{quadr}(0, \Delta/2, J) \qquad \text{(point-matching)}$$
$$[\mathbf{w}, \mathbf{x}] = \text{quadr}(0, \Delta, J) \qquad \text{(Galerkin)}$$

The impedance kernel $Z(z)$ is a scaled version of the Green's function kernel $G(z)$

$$Z(z) = \frac{j\eta}{2\pi}\, G(z)$$

We will consider both the exact and the approximate thin-wire kernels. For an antenna of radius a, the approximate kernel $G(z)$ is defined as follows (see Eq. 21.3.5):

$$G_{\text{approx}}(z) = \frac{e^{-jkR}}{R}, \quad R = \sqrt{z^2 + a^2}$$

The exact kernel is defined by (see Eq. 21.1.2, for $\rho = a$):

$$G_{\text{exact}}(z) = \frac{1}{2\pi}\int_0^{2\pi} \frac{e^{-jkR}}{R}\, d\phi, \quad R = \sqrt{z^2 + 4a^2 \sin^2\frac{\phi}{2}}$$

A useful representation of the exact kernel is in terms of the elliptic integral of the first kind, $K(\kappa)$, and the Jacobian elliptic function $\text{dn}(z, \kappa)$ (see the Wilton-Champagne paper):

$$G_{\text{exact}}(z) = \frac{2K}{\pi R}\int_0^1 e^{-jkR\,\text{dn}(uK,\kappa)}\, du$$

where

$$K = K(\kappa), \quad \kappa = \frac{2a}{R}, \quad R = \sqrt{z^2 + 4a^2}$$

Using this representation, an accurate computation of the exact and approximate kernels can be made with the function `kernel`, with usage:

```
G = kernel(z,a,'e')    (exact kernel)
G = kernel(z,a,'a')    (approximate kernel)
```

where z is a row vector of z-points and G is the corresponding row vector of values $G(z)$, and the quantities z, a must be entered in units of the wavelength λ.

The exact kernel has a logarithmic singularity at $z = 0$, which follows from the logarithmic singularity of $K(\kappa)$ at $\kappa = 1$:

$$G_{\text{exact}}(z) \simeq \frac{1}{\pi a}\ln\left(\frac{4a}{z}\right)$$

With the help of the function `kernel`, the Hallén impedance matrix Z can be computed by the following program fragment for the point-matching case:

```
L = 0.5; a = 0.005; M=50;           % example values
J = 32;                             % number of quadrature points
D = L/(2*M+1);                      % segment width
f = zeros(1,2*M+1);                 % first row of Z

[w,x] = quadr(0,D/2,J);             % quadrature weights and points

for m=0:2*M,                                            % ker='e' or 'a'
  G = kernel(x-m*D,a,ker) + kernel(x+m*D,a,ker);        % G is a row
  f(m+1) = G * w;                                       % w is column
end

Z = toeplitz(f,f);                     % make it a Toeplitz matrix
Z = j*etac(1)/(2*pi) * Z;              % eta = etac(1) = 377 ohm
```

A number of issues that have been discussed and debated for years regarding the solutions of Hallén's equation are as follows:

1. The approximate kernel is non-singular at $z = 0$. Yet, the numerical solution of Halleń's equation using the approximate kernel does not converge and becomes unusable for increasing N and/or for increasing radius a, whereas the solution based on the exact kernel does converge.
2. In fact, it can be shown that under mild regularity assumptions on $I(z)$, the approximate-kernel Hallén equation for a delta-gap input does not have a solution, whereas the one with the exact kernel does.

3. The input impedance of the antenna, $Z_0 = V_0/I(0)$, for the delta-gap case does not converge to a constant value for the approximate kernel as N increases, but it does so for the exact kernel. Generally, numerical methods get the resistive part of Z_0 fairly accurately, but have a hard time for the reactive part.

4. The solution $I(z)$ for the exact kernel in the delta-gap case has a logarithmic singularity at $z = 0$ of the form:

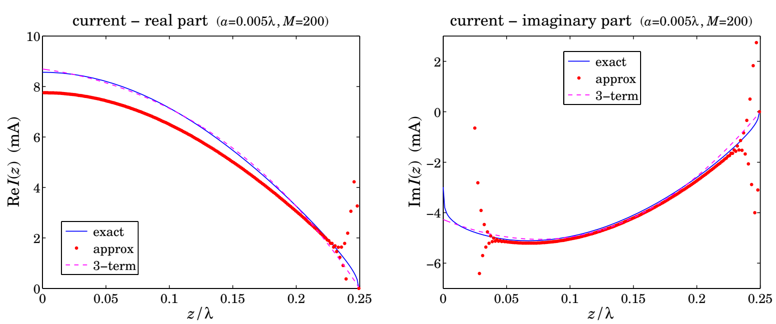
$$I(z) \simeq -j\,\frac{4kaV_0}{\eta}\,\ln\bigl(k|z|\bigr)\,, \quad z \simeq 0$$

   Therefore, one may wonder if the numerical solutions have any use. However, this logarithmic singularity is confined in a very narrow range around $z = 0$ and for all other values of z, the exact-kernel solution is accurate and useful.

5. King's empirical three-term approximation for the current is very accurate (except in the immediate vicinity of the logarithmic singularity at $z = 0$), if fitted to the exact-kernel solution. The three-term approximation can in turn be used to predict the far-field radiation pattern of the antenna.

With the above preliminaries, please carry out the following computer experiments that illustrate the above remarks and the properties of the numerical solutions. Only the point-matching method will be considered—the Galerkin method yielding comparable results.

(a) Consider a dipole antenna of length $l = 0.5\lambda$ and radius $a = 0.005\lambda$. For each of the values $M = 20, 50, 100, 200$, solve Hallén's equation for a delta-gap input with voltage $V_0 = 1$ volt using both the exact and the approximate kernels. Plot the real and imaginary parts of the current $I_m = I(z_m)$ versus z_m over the right half of the antenna, that is, $0 \le z_m \le h$, where $h = l/2$.



(b) King's three-term approximation, fits the antenna current to the following sum of sinusoidal terms, each vanishing at the antenna endpoints $z = \pm h$:

$$I_s(z) = A_1\bigl(\sin k|z| - \sin kh\bigr) + A_2\bigl(\cos kz - \cos kh\bigr) + A_3\bigl(\cos(kz/2) - \cos(kh/2)\bigr)$$

Do a least-squares fit of this expression to the computed current samples I_m of the exact kernel, that is, find the coefficients A_1, A_2, A_3 that minimize the error squared:

$$\mathcal{J} = \sum_{m=-M}^{M} \bigl|I_s(z_m) - I_m\bigr|^2 = \min$$

Then, place the evaluated points $I_s(z_m)$ on the same graphs as in part (a). Discuss how well or not the three-term approximation fits the exact-kernel and the approximate-kernel current.

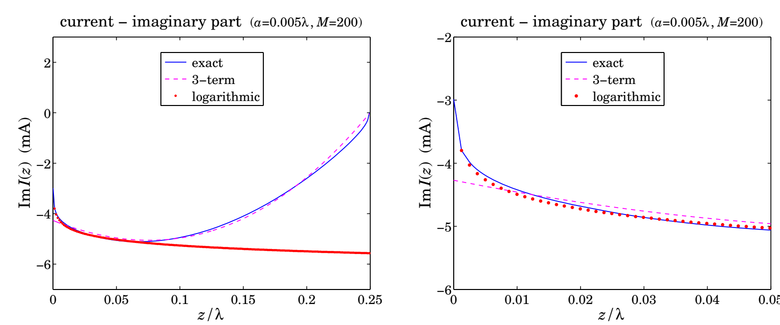
Repeat by using a two-term approximation, that is, setting $A_3 = 0$ and minimizing the above error criterion only with respect to A_1, A_2. Discuss how well or not the two-term approximation fits the exact-kernel and the approximate-kernel current.

(c) To illustrate the logarithmic singularity near $z = 0$, evaluate the limiting expression at the points z_m, $m = 1, 2, \dots, M$, for $M = 200$ (the point $z_0 = 0$ is to be skipped):

$$I_{\log}(z_m) = -j\,\frac{4kaV_0}{\eta}\,\ln\bigl(k|z_m|\bigr) + \text{const.}$$

Adjust the constant so that this expression agrees with the exact-kernel current at the point z_1, that is, $I_{\log}(z_1) = I_1$. Then, plot the imaginary parts of I_m and $I_{\log}(z_m)$ versus z_m. An example graph and its zoomed version are shown at the top of the next page.

(d) Repeat parts (a–c) for the antenna radius $a = 0.001$ and then for $a = 0.008$. Discuss the effect of changing the radius on the quality of the solution, both for the exact and the approximate kernel cases.



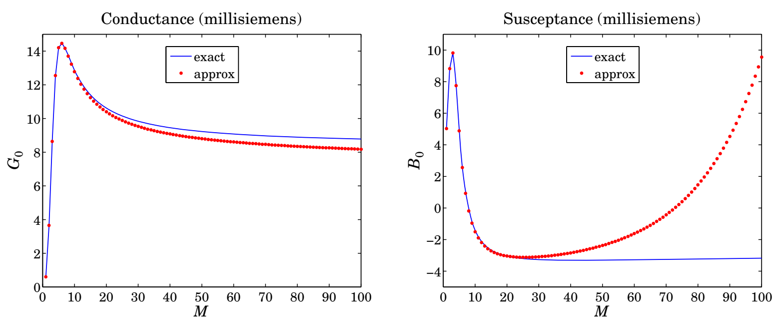
(e) Repeat parts (a–d) for the antenna length $l = 1.0\lambda$. Comment on the success of the exact versus approximate kernel calculations versus the parameters l, a, M.

(f) For each value of M and current solution I_m, $-M \le m \le M$, the input impedance of the antenna can be calculated from the center sample I_0, that is, $Z_0 = V_0/I_0$. Similarly, the input admittance is:

$$Y_0 = \frac{1}{Z_0} = \frac{I_0}{V_0} = G_0 + jB_0$$

where G_0, B_0 are its real and imaginary parts, that is, the input conductance and susceptance.

For each of the values $M = 1, 2, \dots, 100$, calculate the corresponding conductance and susceptance, $G_0(M), B_0(M)$, using the exact and the approximate kernels and plot them versus M. Use the length and radius $l = 0.5\lambda$ and $a = 0.005\lambda$.



This is a time-consuming question. It requires that you solve the Hallén equation for each value of M for the exact and approximate kernels and pick the center value I_0. Discuss the convergence properties of the exact versus the approximate kernel calculation.