332:345 – Linear Systems & Signals – Fall 2009

$F(s)$	$f(t)$	$F(z) = F^*(s)$
$\frac{1}{s}$	$u(t)$	$\frac{1}{1-z^{-1}}$
$\frac{1}{s^2}$	$tu(t)$	$\frac{Tz^{-1}}{(1-z^{-1})^2}$
$\frac{1}{s^3}$	$\frac{1}{2}t^2u(t)$	$\frac{T^2z^{-1}(1+z^{-1})}{2(1-z^{-1})^3}$
$\frac{1}{s+a}$	$e^{-at}u(t)$	$\frac{1}{1-z^{-1}e^{-aT}}$
$\frac{1}{(s+a)^2}$	$te^{-at}u(t)$	$\frac{Te^{-aT}z^{-1}}{(1-z^{-1}e^{-aT})^2}$
$\frac{1}{(s+a)^3}$	$\frac{1}{2}t^2e^{-at}u(t)$	$\frac{T^2e^{-aT}z^{-1}(1+z^{-1}e^{-aT})}{2(1-z^{-1}e^{-aT})^2}$
$\frac{1}{s(s+a)}$	$\frac{1}{a}(1-e^{-at})u(t)$	$\frac{(1-e^{-aT})z^{-1}}{a(1-z^{-1})(1-z^{-1}e^{-aT})}$
$\frac{1}{s^2(s+a)}$	$\frac{1}{a}tu(t)-\frac{1}{a^2}(1-e^{-at})u(t)$	$\frac{z^{-1}(aT+e^{-aT}-1)+z^{-2}(1-e^{-aT}-aTe^{-aT})}{a^2(1-z^{-1})^2(1-z^{-1}e^{-aT})}$
$\frac{1}{(s+a)(s+b)}$	$\frac{e^{-at}-e^{-bt}}{b-a}u(t)$	$\frac{(e^{-aT}-e^{-bT})z^{-1}}{(b-a)(1-z^{-1}e^{-aT})(1-z^{-1}e^{-bT})}$
$\frac{1}{s+a-j\omega_0}$	$e^{-at}e^{j\omega_0 t}u(t)$	$\frac{1}{1-z^{-1}e^{-aT}e^{j\omega_0 T}}$
$\frac{\omega_0}{(s+a)^2+\omega_0^2}$	$e^{-at}\sin(\omega_0 t)u(t)$	$\frac{z^{-1}e^{-aT}\sin(\omega_0 T)}{1-2z^{-1}e^{-aT}\cos(\omega_0 T)+z^{-2}e^{-2aT}}$
$\frac{s+a}{(s+a)^2+\omega_0^2}$	$e^{-at}\cos(\omega_0 t)u(t)$	$\frac{1-z^{-1}e^{-aT}\cos(\omega_0 T)}{1-2z^{-1}e^{-aT}\cos(\omega_0 T)+z^{-2}e^{-2aT}}$

$f(n)$	$F(z)$
$\delta(n-D)$	z^{-D}
$u(n)$	$\frac{1}{1-z^{-1}}$
$nu(n)$	$\frac{z^{-1}}{(1-z^{-1})^2}$
$n^2u(n)$	$\frac{z^{-1}(1+z^{-1})}{(1-z^{-1})^3}$
$a^nu(n)$	$\frac{1}{1-az^{-1}}$
$na^nu(n)$	$\frac{az^{-1}}{(1-az^{-1})^2}$

Padé approximations of a delay:

$$e^{-\tau s} \approx \frac{1-\tau s/2}{1+\tau s/2}$$

$$e^{-\tau s} \approx \frac{1-\tau s/2+\tau^2 s^2/12}{1+\tau s/2+\tau^2 s^2/12}$$

$$e^{-\tau s} \approx \frac{1-\tau s/2+\tau^2 s^2/10-\tau^3 s^3/120}{1+\tau s/2+\tau^2 s^2/10+\tau^3 s^3/120}$$

Hermite interpolation formula:

$$P(t_1)=a_1\,,\quad \dot{P}(t_1)=b_1\,,\quad P(t_2)=a_2\,,\quad \dot{P}(t_2)=b_2$$

$$P(t) = \left(\frac{t-t_2}{T}\right)^2\left[a_1 + (Tb_1+2a_1)\left(\frac{t-t_1}{T}\right)\right] + \left(\frac{t-t_1}{T}\right)^2\left[a_2 + (Tb_2-2a_2)\left(\frac{t-t_2}{T}\right)\right]\,,\quad T = t_2 - t_1$$