Chapter 4 studies two-dimensional multivalued differential automata. The main result of this chapter is an analog of the Poincaré–Bendixon Theorem for hybrid systems. It provides conditions for a hybrid system in the plane to have a non-chaotic, eventually periodic behavior. As an application, the authors give a detailed analysis of two specific switched flow systems.

Chapters 5 and 6 deal with hybrid dynamical systems with constant derivatives. This class of systems is related to the one considered in Chapter 2, although the exact relationship unfortunately does not seem to be explained anywhere. Chapter 5 develops general criteria for existence and stability of limit cycles in such systems. Chapter 6 illustrates the theory by studying several specific classes of switched flow systems.

The investigation of switched flow systems is continued in Chapters 7 and 8. Chapter 7 considers single server switched flow networks and provides conditions for existence of a single globally attracting limit cycle. Chapter 8 is concerned with multiple server switched flow networks with time-varying arrival rates. The problem addressed here is design of feedback policies which provide desirable regular behavior in such systems. A simple algebraic necessary and sufficient condition for such a feedback policy to exist is obtained.

Finally, Chapter 9 describes seven open problems. These problems range from the theoretical one of extending the theory of this book to differential automata with nonlinear right-hand side to the problem of proving a specific conjecture for switched server systems with more than three buffers.

The book presents original results due to the authors, with complete proofs provided. The reviewer found the book well organized and methodically written. A particular nice feature of the book is that footnotes are included in the statements of the main results to help the reader quickly find the relevant definitions.

The application domains discussed extensively in the book are queueing networks and manufacturing systems. This book is an essential reading for anyone interested in applying hybrid system methods to these fields. The theory described in the book is likely to be useful for other application areas, such as power systems.

The reviewer also recommends this book to all students and researchers who want to follow a careful development of a general definition of a hybrid system, supplemented with helpful and realistic examples. On the other hand, due to its limited scope, this is not the right book for someone who is looking for an overview of techniques and problems in the general area of hybrid systems and control. For example, one of many important and interesting topics not covered in the book is stability of equilibria for hybrid systems. The book is also probably not suitable as a textbook, except perhaps for an advanced course with a somewhat narrow focus. To get a broader perspective, the reader will need to consult other sources, such as the book by van der Schaft and Schumacher (2000).

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Optimal control of singularly perturbed linear systems and applications

Zoran Gajić and Myo-Taeg Lim; Marcel Dekker, New York, 2001, ISBN: 0-8247-8976-8

Singular perturbations arise in systems whose dynamics have sufficiently separate slow and fast parts. They also are seen due to weak coupling of subsystems, and may be induced under feedback when applying high-gain control, or "cheap" optimal control. Since the late 1960s, extensive research on singularly perturbed control systems, and parallel efforts on singularly perturbed ordinary differential equations, has yielded a rich and powerful body of results. The book under review (Gajić & Lim, 2001) is a compact monograph reporting recent developments in the optimal control of linear, time-invariant, singularly perturbed systems. The emphasis is on situations in which low-order approximations are inadequate. The authors develop recursive methods to solve with high accuracy exactly decoupled pure-slow and pure-fast optimal regulation and filtering problems. They implement a number of useful variations on their approach, including continuous- and discrete-time systems, H_2 and H_∞ optimization, and a class of decentralized control problems. To fix ideas, small examples are sprinkled throughout. A resourceful reader, armed with a basic knowledge of singular perturbations and a capable matrix manipulation software package, should be designing controllers in a matter of days.

Although a brief history of the field is provided, along with a useful list of references, this is not the place to begin a study of singular perturbations. Before settling down with this book, the novice would do well to get a broader introduction from, for example, the excellent collection of fundamental papers in Kokotovic and Khalil (1986), the text by Kokotovic, Khalil, and O'Reilly (1986), or the introductory articles contained in Kokotovic, Bensoussan, and Blankenship (1986a). In particular, the book by Kokotovic, Khalil and O'Reilly (1986) is accessible and comprehensive, contains numerous exercises and examples, and has recently been reprinted by SIAM in the Classics in Applied Mathematical series. In addition to providing more detailed introductory material, these also discuss a much wider variety of topics, including time-varying, nonlinear, and large-scale systems. The first author of Gajić and Lim (2001) has co-authored an earlier monograph (Gajić & Shen, 1993), that includes some subjects omitted from the present volume, such as output feedback, applications to differential games and bilinear systems, and extensive coverage of weakly coupled systems.

The title (Gajić & Lim, 2001) is from a series whose stated goal is to make newly developed tools of control engineering accessible to the practitioner. In this respect the present volume is not entirely successful. One might expect a relaxed and expository tone; in contrast the book mainly reads as a lightly edited series of journal papers. Certain signature phrases appear over and over ("the celebrated Chang transformation"). The repetitive style provides the reader with a frequent sense of déjà vu, even though very little of the material is actually superfluous. On the positive side, the various chapters are completely self-contained. To truly aid the practicing control engineer some space might have been given over to implementation issues. Presumably the authors have developed a suite of software implementing their algorithms. Matlab-like pseudo-code would have been welcome. A CD-ROM with working routines, or a link to a website where such routines could be downloaded, would have been even better. On the other hand, the methods described in Gajić and Lim (2001) appear to be of true practical value. Many results on singularly perturbed systems are asymptotic—holding in the limit as the perturbation parameter becomes small. Because the size of this parameter is typically not something the control engineer can adjust, such results may be of primarily theoretical interest. In contrast, the approach of the authors is recursive, and the number of iterations can be selected to produce a satisfactory result.

To convey the benefits of the methods of Gajić and Lim (2001), consider the problem of finding the optimal state feedback gains solving the infinite horizon linear-quadratic regulator (LQR) problem (Kwakernaak & Sivan, 1972) for the following singularly perturbed linear, time-invariant system:

$$\dot{x}_1 = A_{11}x_1 + A_{12}x_2 + B_1u, \tag{1}$$

$$\varepsilon\dot{x}_2 = A_{21}x_1 + A_{22}x_2 + B_2u, \tag{2}$$

$$y = C_1x_1 + C_2x_2. \tag{3}$$

Here ε is the perturbation parameter and system (1)–(3) is said to be in standard form if A_{22} is invertible. Such a system might arise, for example, from a natural time-scale separation between the "slow" states x_1 and the "fast" states x_2. For convenience we refer to A, B, and C—the system matrices corresponding to the state equation for $x = [x_1^{\mathrm{T}}, x_2^{\mathrm{T}}]^{\mathrm{T}}$. Let us now compare three representative options for solving this problem:

(1) The structure of (1)–(3) may be ignored, and the full regulator algebraic Riccati equation (ARE) solved (Kwakernaak & Sivan, 1972). Considering (1) and (2) as being in generalized state-space form, $E\dot{x} = Ax + Bu$, the ill-conditioning of $E = \mathrm{diag}(I_{n_s}, \varepsilon I_{n_f})$ ensures the ill-conditioning of the ARE (Arnold III & Laub, 1984), leading to numerical difficulties. Further, we will have missed a chance to reduce the size and complexity of our controller. This is a particularly important consideration for large systems.

(2) The procedure given in Chow and Kokotovic (1976) provides well-conditioned computation of a reduced-order controller. First, a "pure-slow" subsystem is obtained by setting $\varepsilon = 0$ in (2), and solving for the resulting value of x_2: $A_{21}\bar{x_1} + A_{22}\bar{x_2} + B_2\bar{u} = 0 \Rightarrow \bar{x_2} = -A_{22}^{-1}(A_{21}\bar{x_1} + B_2\bar{u})$. Define the pure-slow system variables to be $x_s = \bar{x_1}$, $u_s = \bar{u}$, $y_s = \bar{y} = C_1\bar{x_1} + C_2\bar{x_2}$, and the pure-fast system variables to be $x_f = x_2 - \bar{x_2}$, $y_f = y - y_s$, $u_f = u - u_s$. Set $A_0 = A_{11} - A_{12}A_{22}^{-1}A_{21}$, $B_0 = B_1 - A_{12}A_{22}^{-1}B_2$, $C_0 = C_1 - C_2A_{22}^{-1}A_{21}$ and $D_0 = -C_2A_{22}^{-1}B_2$. Then the approximately decoupled subsystems result:

$$\dot{x}_s = A_0x_s + B_0u_s, \qquad \varepsilon\dot{x}_f = A_{22}x_f + B_2u_f,$$

$$y_s = C_0x_s + D_0u_s, \qquad y_f = C_{22}x_f.$$

The decoupling is only approximate because the pure-slow dynamics as defined hold exactly only for $\varepsilon = 0$. Note that a factor of $1/\varepsilon$ scales all components on the right-hand side of the pure-fast subsystem; thus the potential conditioning problems are resolved. Regulator AREs are solved independently for the two subsystems to obtain slow and fast feedback gains, K_s and K_f.

The *composite control*, $u = K_s x_s + K_f x_f = K_1 x_1 + K_2 x_2$,is an $O(\varepsilon^2)$ approximation to the true optimal value (Chow & Kokotovic, 1976). This is a valuable result. Two smaller computations are preferred to a single large one, and the resulting compensator complexity is reduced. The AREs are well conditioned, and may be solved by standard software. However, the near optimality of the solution (and in fact, even stability) is guaranteed only asymptotically, for "sufficiently small" ε. If this accuracy is insufficient for a particular ε, the designer must seek to refine the calculation, as ε is prescribed. Unfortunately it is not clear how to proceed further using this approach.

(3) A method for precisely such refinement is found in Gajić and Lim (2001, Chapter 2). The authors make extensive use of a transformation due to Chang (1972) that *exactly* decouples the slow and fast subsystems. Rather than apply the Chang transformation to (1) and (2) directly, the authors first construct the Hamiltonian form of the optimal closed-loop system, with new slow states consisting of x_1 and the slow costates, and new fast states consisting of x_2 augmented by the fast costates. Their results are motivated and enabled by the observation that the Hamiltonian system maintains its Hamiltonian structure under the exact decoupling transformation. Manipulation of the decoupled pure-slow and pure-fast Hamiltonian systems results again in two AREs, which may be solved for the pure-slow and pure-fast state feedback gains. These differ from those obtained in Chow and Kokotovic (1976) in two important ways. The first is that the corresponding composite control is *exactly* the optimal control. The second is that the decoupled regulator AREs are nonsymmetric.

The authors detail two ways of solving the decoupled, nonsymmetric regulator AREs. The first is by Newton's method, in which one iteratively solves a series of recursive Sylvester equations, starting with a reasonable initial guess (the authors suggest the low-order approximation of Chow and Kokotovic (1976) for this purpose) until the desired accuracy is achieved. Numerical computation of the Chang transformation to comparable accuracy is also required. The authors supply an algorithm based on Newton's method by which this may be accomplished. The second means of solving the nonsymmetric AREs, described in Gajić and Lim (2001, Chapter 7), involves the computation of the eigenvectors of a matrix formed from the matrix coefficients of the ARE. (The Chang transformation can also be computed in a similar way.) The authors recommend this method when ε is relatively large, but it is vulnerable to ill conditioning if ε is small. They go so far as to consider its application when $\varepsilon = 1$, and the system is regular. This seems like a stretch, since many of the key results applied in the book would not generally apply in this circumstance (for example, the existence of a solution to the equations defining the Chang transformation is guaranteed only under the assumption that ε is sufficiently small). We see that a major contribution of Gajić and Lim (2001) is a means by which a decoupled design may be salvaged if, for example, the results of Method 2 are unsatisfactory.

In addition to the results on regulation described above, Chapter 2 presents a dual development for computing a decoupled Kalman filtering, and a decoupled formulation of the finite-horizon open-loop optimal control problem. The authors combine the infinite-horizon state regulator with the Kalman filter to obtain a very elegant, almost entirely decoupled, solution to the linear-quadratic Gaussian optimal control problem. An earlier version, presented in Gajić and Shen (1993), required both the pure-fast and pure-slow state estimates in the innovation process. The version now presented contains the pure-slow state estimates only in the innovation process for the pure-slow Kalman filter, and likewise the pure-fast state estimates only in the innovation process for the pure-fast Kalman filter. Thus the only remaining interaction between the slow and fast compensators occurs in the control u, which contains both slow and fast components and is fed forward through the state estimators. It is difficult to see how this remaining coupling can be avoided. Chapter 3 treats discrete-time systems. The optimal finite- and infinite-horizon regulator problems are solved, and the discrete Kalman filter constructed. Interestingly, the nonsymmetric decoupled pure-slow and pure-fast Riccati equations that result are of the type associated with continuous-time systems, rather than of the discrete-time type associated with optimal control of the full system. Once again an initial guess is found by solving symmetric AREs for the fast subsystem (of discrete type) and slow subsystem (of continuous type). Iterative refinement is again by solution of Sylvester equations.

Chapter 4 presents optimal control and filtering results for a type of multimodeling structure (Kokotovic, Bensoussan, & Blankenship, 1986a) in which a slow subsystem is strongly coupled to two, weakly coupled, fast subsystems. Here the decomposition is into one pure-slow, and two pure-fast subsystems. The perturbation parameters associated with the two pure-fast subsystems may be different.

Chapter 5 extends the results of Chapter 2 to basic H_∞ suboptimal control and filtering. The regulator problem considered is to find the observer-based controller $K(s)$ such that the infinity norm of the closed-loop transfer function from disturbance to regulated outputs is less than some specified value, γ. As the authors describe, in a well-written introductory section, the solution of this problem is known to be given by the solution to two AREs, one specifying the state feedback gains, the other used to construct the observer gains. These AREs are in turn associated with Hamiltonian systems describing state and costate dynamics. Once the Hamiltonian matrices have been constructed, the procedure exactly follows Chapter 2. The observer that appears in the optimal controller is not a Kalman filter. An H_∞ analog to the Kalman filter may be defined, and the resulting filter gains are known to be described by the solution to an ARE. Again the singularly perturbed case is solved by constructing the resulting Hamiltonian matrix, applying the decoupling Chang transformation, and iteratively solving the resulting pure-slow and pure-fast nonsymmetric AREs.

Chapter 6 addresses situations in which the time scale separation is not inherent in the system, but is induced under feedback due to high gain control, optimal regulation with low control weighting, or optimal estimation with small measurement noise. As in Chapter 2, results include finite- and infinite-horizon performance optimization of linear, time-invariant dynamics. A filtering problem is also solved, applicable to problems with small sensor noise. Although the high-gain and cheap control problems differ slightly in formulation, with the perturbation parameter entering the former through the B matrix and the latter through the control u, the resulting full system AREs corresponding to the optimal control are identical. Hence they are treated in a unified fashion. Similarly, the small noise problem is dual to the cheap control problem, and is also handled in this framework. To this point the results closely parallel Chapter 2. Chapter 6 closes by examining a sampled-data system, in which the role of the perturbation parameter is played by the sampling time. The sampling time also appears in the performance criterion, where its square multiplies the control cost. Thus, this is a class of discrete-time cheap control problems. First-order approximations to the solutions are given by the solutions to symmetric AREs for the pure-slow (continuous type) and pure-fast (discrete type) subsystems. The authors recommend Newton's method for subsequent refinement, but do not supply the recursion relations.

As mentioned earlier, Chapter 7 describes an alternative technique for finding the nonsymmetric ARE solutions and the Chang transformation based on eigenvector computations. This approach may be numerically ill conditioned for small ε.

Finally, Chapter 8 discusses a number of interesting topics, and suggests ways in which the techniques of the preceding chapters may be extended. Topics include nonstandard systems, in which the matrix A_{22} is singular; finite-horizon problems, in which the Riccati equations are differential instead of algebraic; and slow–fast integral manifold decomposition, which provides a nonlinear version of the Chang decomposition, and which also applies to the linear time-varying case. Of these, the presentation of nonstandard systems is reasonably complete; the remaining discussions are mainly suggestive of directions that future research may take.

Who should own this book? Undoubtedly, any serious researcher in the area of optimal control of singularly perturbed systems. The rest of us would benefit from consulting it while attempting to design such a controller. However, implementing the authors' algorithms requires some potentially nontrivial software development. For LQR or LQG design some of the older methods, as vividly described by Kokotovic, Khalil and O'Reilly (1986), remain of interest, unless demonstrated in a particular application to be insufficiently accurate. However for higher-order design, as well as for H_∞ control of singularly perturbed systems, or any of the other applications detailed above, this book would be an invaluable and indispensable resource.

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