4.7 MATLAB Laboratory Experiment

Part 1. (a) Using the MATLAB function `poly`, find the characteristic polynomial for an F-8 aircraft whose continuous-time model is given in Section 3.5.2. Use the Routh–Hurwitz test to examine the stability of the polynomial obtained. Check the answer obtained by the MATLAB function `eig`, that is, by finding the system eigenvalues.

(b) For a fifth-order continuous-time model of an industrial reactor given in Problem 3.23, examine the stability by directly finding system eigenvalues and by using the Lyapunov method. Hint: Use $\mathbf{Q} = \mathbf{I}_5$.

Part 2. (a) A discrete-time model of a chemical plant (Gomathi *et al.*, 1980; Gajić and Shen, 1993) is given by the following system matrix

$$\mathbf{A} = \begin{bmatrix} 0.9541 & 0.0196 & 0.0036 & 0.0007 & 0.0002 \\ 0.4085 & 0.4132 & 0.1608 & 0.0447 & 0.0120 \\ 0.1222 & 0.2633 & 0.3615 & 0.1593 & 0.1238 \\ 0.0411 & 0.1286 & 0.2721 & 0.2144 & 0.4098 \\ 0.0013 & 0.0058 & 0.0188 & 0.0362 & 0.9428 \end{bmatrix}$$

Using MATLAB, find its characteristic polynomial. Examine the stability of this system by using (a) the eigenvalue approach, (b) Jury's test, and (c) Lyapunov's method.

(b) Repeat the analysis done in Part 2a for a discrete-time model of a distillation column (Kautsky *et al.*, 1985) given by

$$\mathbf{A} = \begin{bmatrix} 0.9895 & 0.0056 & 0.0003 & 0.0000 & 0.0000 \\ 0.1172 & 0.8145 & 0.0760 & 0.0056 & 0.0004 \\ 0.0088 & 0.1239 & 0.7502 & 0.1080 & 0.0112 \\ 0.0009 & 0.0180 & 0.1838 & 0.6683 & 0.1508 \\ 0.0000 & 0.0003 & 0.0017 & 0.0133 & 0.9852 \end{bmatrix}$$

Part 3. (a) Use MATLAB and its function `nyquist(num,den,w)` to plot the Nyquist diagram for the following system

$$G(s)H(s) = \frac{100}{(s-1)(s+10)^2}$$

Determine the stability of the closed-loop system using the Nyquist criterion. Hint: Use MATLAB axis scaling function as `axis([-1.4 0.2 -0.6 0.6])`.

(b) Find the gain and phase stability margins by using MATLAB function

```
[Gm,Pm,wcp,wcg]=margin(num,den)
```

for the problem studied in Example 4.24, now parametrized by a constant K, that is

$$G(s)H(s) = \frac{K}{s(s+1)(s+2)}$$

Take $K = 2, 3, 4, 5, 6, 10, 100$ and comment on the results obtained.

Part 4. Use the MATLAB function `conv` to obtain a shifted characteristic polynomial of the characteristic polynomial

$$s^6 + 2s^5 + s^4 + s^3 + 2s^2 + 2s + 1$$

i.e. replace s by $s_1 - 1$. Apply the Routh–Hurwitz method to both polynomials. Find the exact location of the roots of the original characteristic polynomial and comment on the results obtained from the Routh–Hurwitz tests.

Part 5.[7] (a) For the antenna control system whose transfer function is given in Problem 3.1, find the state space form for $K = 1$. Examine system stability by finding the system minimal polynomial. Hint: Use MATLAB to find n_i from formula (4.20).

(b) Justify the answer obtained in (a) by finding and plotting the system response due to the initial condition $\mathbf{x}^T(0) = [1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1]$. Hint: Take `t=0:1:100`, calculate `[y,x,t]=initial(A,B,C,D,x0,t)`, and then use `plot(x)`.

[7] This part is based on the material presented in Subsections 4.2.4–4.2.6.