PID Controllers in Nineties

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Overview

- Purpose: extract the essence of the most recent development of PID control
- Based on the survey of papers (333) in nineties in the following journals:
  - **IEEE Transactions on Automatic Control (23)**
  - **IEEE Transactions of Control Systems Technology (26)**
  - IEEE Transactions on Robotic and Automation (11)
  - IEEE Transactions on Industrial Electronics (2)
  - IEEE Control Systems Magazine (4)
  - **IFAC Automatica (59)**
  - **IFAC Control Engineering Practice (29)**
  - **International Journal of Control (20)**
  - International Journal of Systems Science (2)
  - International Journal of Adaptive Control and Signal Processing (2)
  - **IEE Proceedings - Control Theory and Applications (30)**

Overview

- Based on the survey of papers (333) in nineties:
  - Journal of the Franklin Institute (5)
  - Control and Computers (1)
  - Computing & Control Engineering Journal (3)
  - Computers and Chemical Engineering (1)
  - **AIChE Journal (16)**
  - Chemical Engineering Progress (2)
  - Chemical Engineering Communications (1)
  - **Industrial and Engineering Chemistry Research (55)**
  - **ISA Transactions (21)**
  - **Journal of Process Control (13)**
  - Transactions of ASME (1)
  - ASME Journal of Dynamic Systems, Measurements and Control (3)
  - Electronics Letters (2)
  - Systems & Control Letters (1)

Paper Classification

- Ziegler-Nichols based PIDs (10)
- Frequency domain based PIDs (22)
- Relay based PIDs (29)
- Optimization methods based PIDs (20)
- Internal Model Control PIDs (15)
- Robust PID controllers (30)
- Nonlinear PIDs (12)
- Adaptive PIDs (28)
- Anti-windup techniques (13)
- Neural Network/Fuzzy Logic based PIDs (34)
- PID control of Distributed Systems (3)
- Multivariable PIDs (29)
- Applications of PID controllers (56)

Forms of PID controller

$$u = K_p\left(e + \frac{1}{T_i}\int e\,dt + T_d\frac{de}{dt}\right)$$

$$e = y_r - y$$

Standard form

$$G_s(s) = K_p\left(1 + \frac{1}{T_i s} + T_d s\right)$$

$$G_c(s) = K'\left(1 + \frac{1}{T_i' s}\right)(1 + T_d' s)$$

Cascade form

$$G_c(s) = k + \frac{k_i}{s} + k_d s$$

Parallel form

$$u_c = k_c\left(e + \frac{1}{T_i}\int e\,dt - T_d\frac{dy_f}{dt}\right)$$

$$e = y_r - y$$

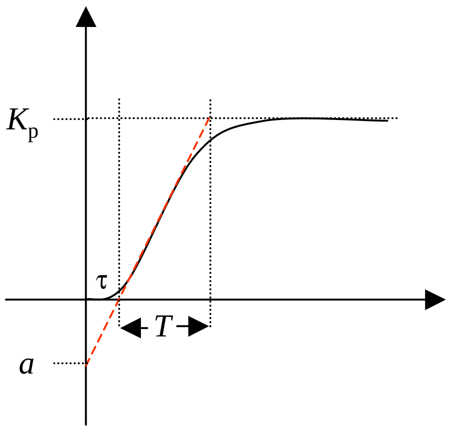
$$y_f = \frac{1}{1 + sT_d / N}\,y$$

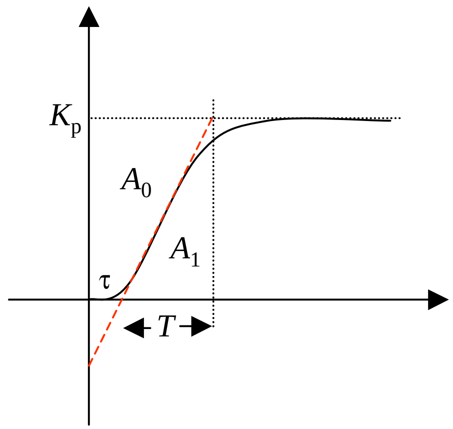
Filtered derivative term

$$u_c = k_c\left[(\beta y_r - y) + \frac{1}{T_i}\int e\,dt - T_d\frac{dy_f}{dt}\right]$$

Weighted setpoint form

Modeling (step response)



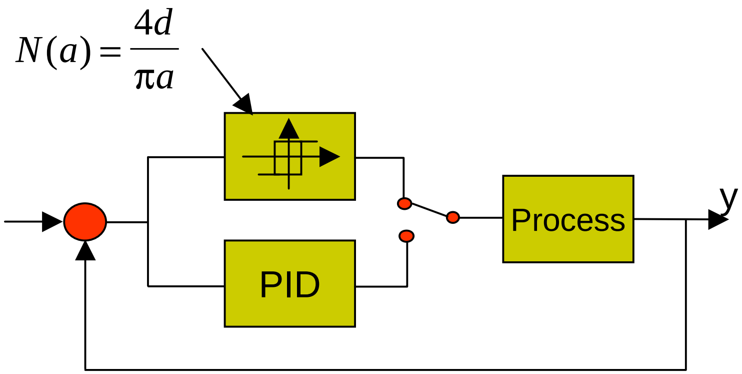


$$G(s) = \frac{a}{s\tau} e^{-\tau s} \qquad G(s) = \frac{K_p e^{-s\tau}}{1 + sT}$$

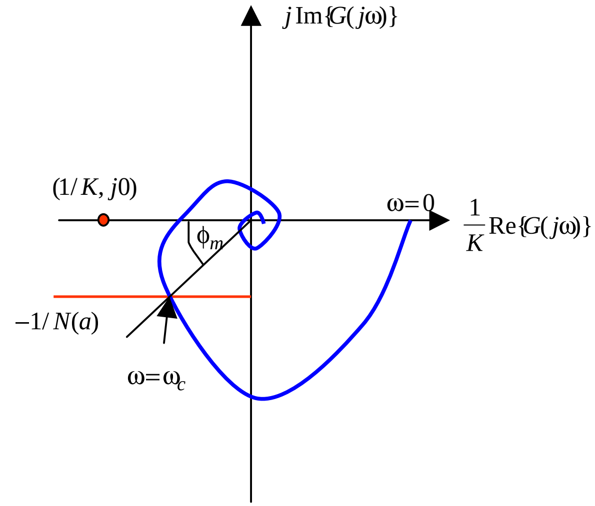
$$T + \tau = \frac{A_0}{K_p}$$

$$T = \frac{A_1}{K_p} e^1$$

Modeling (frequency domain)



(a) Relay Excitation



(b) Correlation Method:

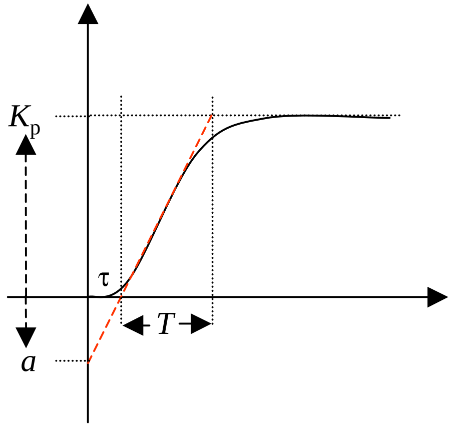
- use PRBS test signal u(t),
- measure y(t),
- find cross-correlation function between u(t) and y(t)
- compute the impulse response g(t)
- transform g(t) to G(s) and find the parameters of the model

Tuning Techniques

- Ziegler-Nichols (10)
- Frequency domain tuning (22)
- Relay based tuning (29)
- Tuning using optimization (20)
- Internal model control tuning (15)
- Other tuning techniques (30)

Ziegler-Nichols Tuning

- Originated by work of Ziegler and Nichols, 1942
- Still in broad industrial use
- Several improvements reported
- Controllers tuned by this method tend to have large overshoot
- Two methods - time and frequency domain based
- Improvements reported (DePoor & I'Malley, 1989; Manz & Taconi, 1989; Chen, 1989; Hang & Sin, 1991, Astrom *et al,* 1992; Cox *et al,* 1997)

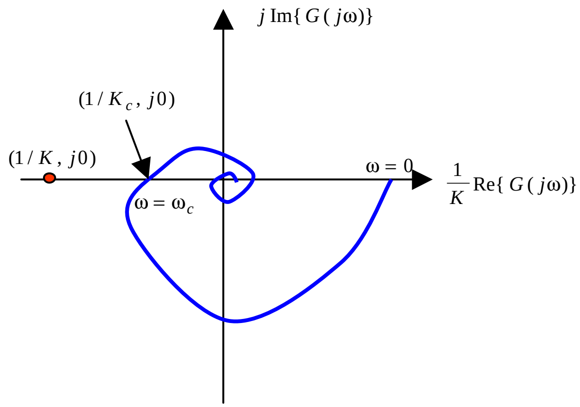


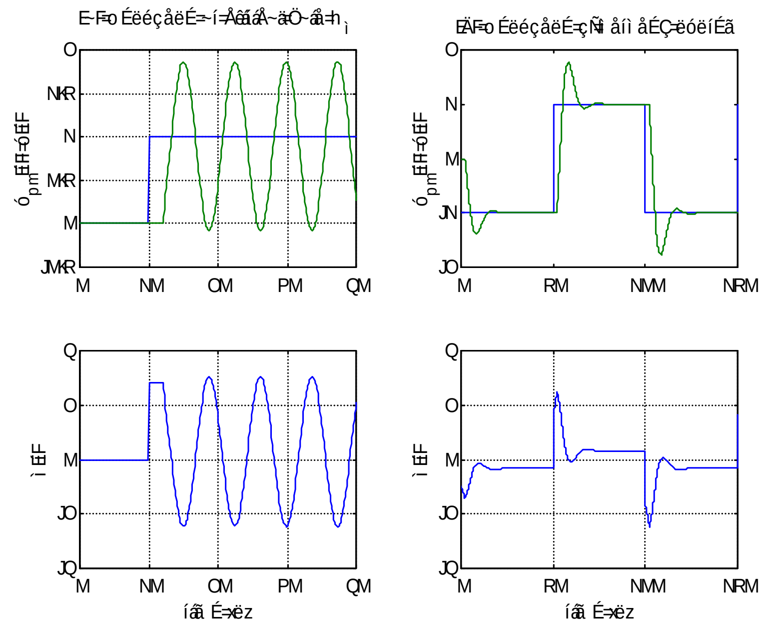
$$u_c = k_c\left[(\beta y_r - y) + \frac{1}{T_i}\int e\,dt - T_d\,\frac{dy_f}{dt}\right]$$

$$0 < \beta < 1$$

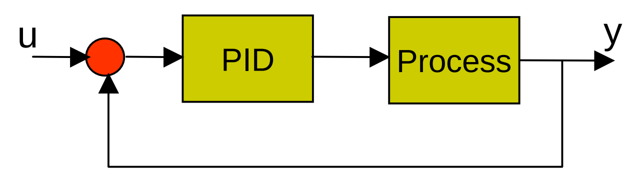
Controller	K	T_i	T_d
P	1/a		
PI	0.9/a	3L	
PID	1.2/a	2L	L/2

Ziegler-Nichols Freq. Response





	PID	PI
Proportional gain	$k_c = 0.6k_u$	$k_c = 0.45k_u$
Integral time	$T_i = 0.5t_u$	$T_i = 0.85t_u$
Derivative time	$T_d = 0.125t_u$	



Refined Ziegler-Nichols

Based on normalized parameters:

$$\kappa = k_p k_u$$

$$\Theta = \frac{\theta_a}{T_p} = \frac{a}{k_p}$$

Refined Ziegler-Nichols formulae for PID control

	PID
Large normalized Process gain or small normlized dead time $2.25 < \kappa < 15;\ 0.16 < \Theta < 0.57$ $\beta = \frac{15-\kappa}{15+\kappa}$ (10% overshoot) $\beta = \frac{36}{27+5\kappa}$ (20% overshoot)	$k_c = 0.6 k_u$
Small normalized process gain or large normalized dead time $1.5 < \kappa < 2.25;\ 0.57 < \Theta < 0.96$ $\mu = \frac{4}{9}\kappa;\ \beta = \frac{8}{17}\left(\frac{4}{9}\kappa + 1\right)$ (20% overshoot and 10% undershoot)	$T_i = 0.5 \mu t_u$
Derivative time	$T_d = 0.125 t_u$

Frequency Domain Tuning Techniques

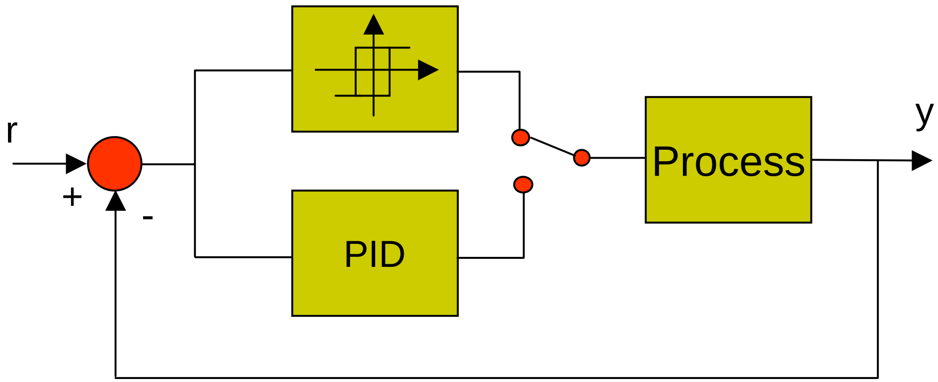
- Variety of the techniques based on desired phase and gain margin, and other frequency response parameters:
  - Hagglund & Astrom 1992;
  - Tyreus & Luyben, 1992;
  - Venkatashankar & Chidambaram, 1994;
  - Wang *et al*, 1995, 1997, 1999;
  - Ho *et al*, 1995,1998;
  - Luyben, 1996, 1998;
  - Khan & Leman, 1996;
  - Poulin & Pomerlau, 1996;
  - Loron, 1997;
  - Shafei & Shenton, 1997;
  - Natarjan & Gilbert, 1997;

Relay Based Tuning Techniques

- Introduced by (Astrom and Hagglund, 1994)
- Considered in many papers
- Relay Tuning considering two-parameter nonlinearity (Friman and Waller, 1995)
- Enhanced relay tuning by using the estimate at the two points of the Nyquist plot (Sung and Lee, 1997)
- Relay tuning that identifies three frequency data sets (Tan *et al., 1996*) using one feedback relay test
- multiple-point frequency response fitting based on relay tuning (Wang *et al.,* 1999)
- Two relays working in parallel (Friman and Waller, 1997)
- A specialized book on relay tuning (Yu, 1999)

Relay Tuning Basics

- Relay tuning (Astrom , Hagglund) is one of the most important methods commercially used



$$1 + N(a,\omega)G(j\omega) = 0 \quad \Rightarrow \quad G(j\omega) = -\frac{1}{N(a,\omega)}$$

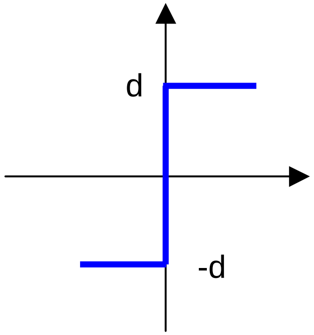
$$N(a) = \frac{4d}{\pi a}$$

$$\mathrm{Re}\{G(j\omega)\} = -\frac{1}{N(a)}, \qquad \mathrm{Im}\{G(j\omega\} = 0$$

$$-\frac{1}{K_c} = -\frac{1}{N(a)} = -\frac{\pi a}{4d} \quad \Rightarrow \quad K_c = \frac{4d}{\pi a}$$

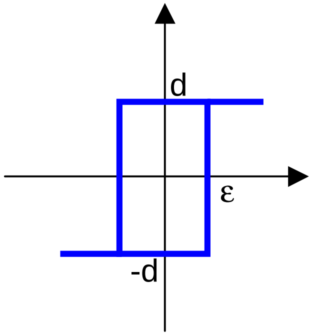
Types of Relays

Ideal Relay



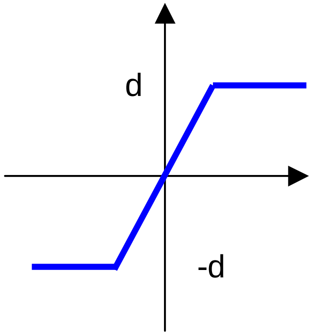
$$N(a) = \frac{4d}{\pi a}$$

Relay with Hysteresis



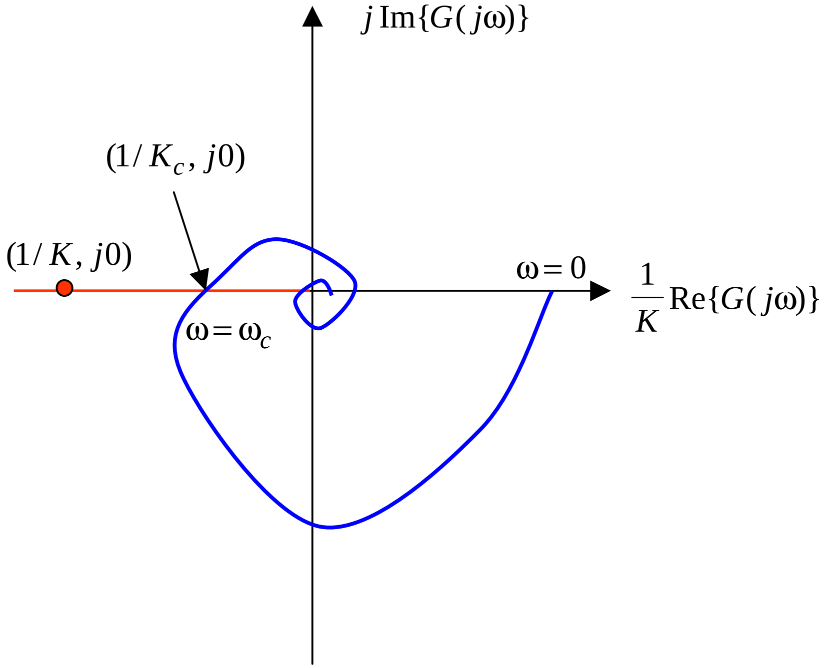
$$N(a) = \frac{4d}{\pi a}\left[\sqrt{a^2 - \varepsilon^2} - j\varepsilon\right]$$

Saturation Relay

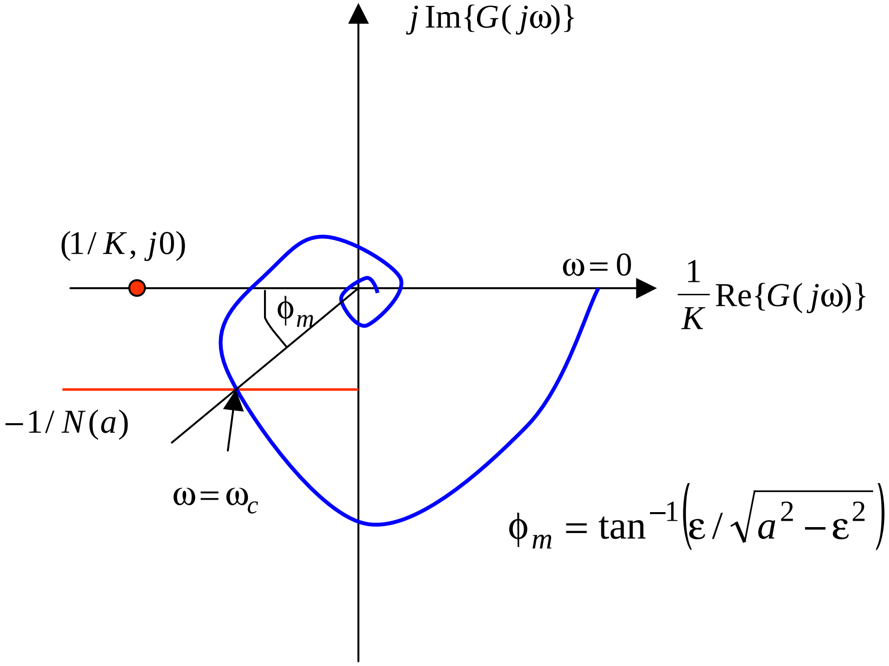


$$N(a) = \frac{2d}{\pi}\left(\frac{1}{\bar{a}}\sin^{-1}\frac{\bar{a}}{a} + \sqrt{\frac{a^2 - \bar{a}^2}{a^2}}\right)$$

Limit Cycle Parameters

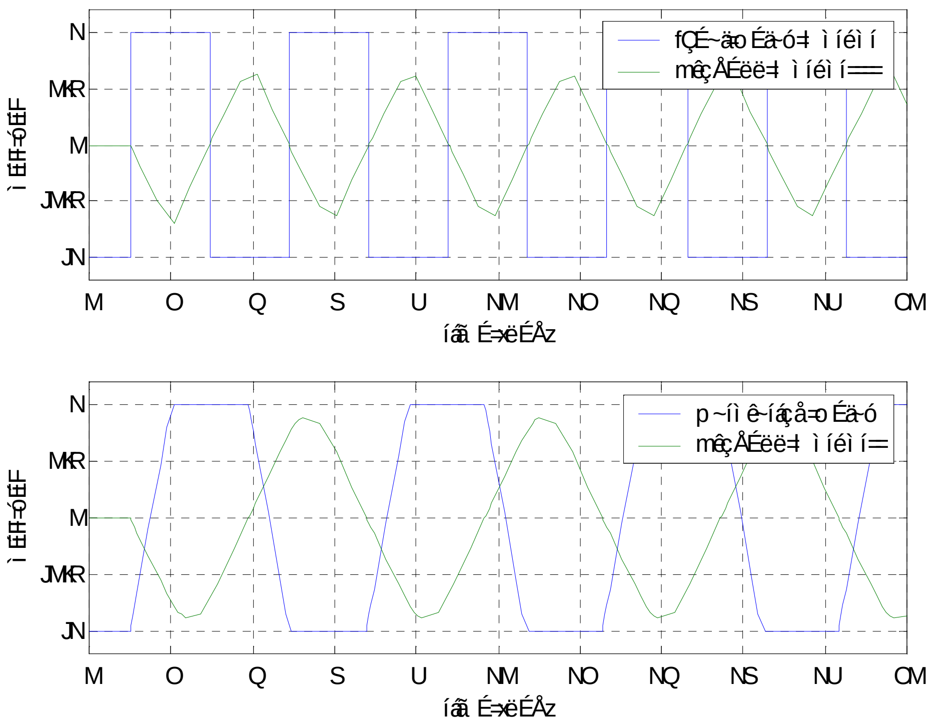


Ideal/Saturation Relay



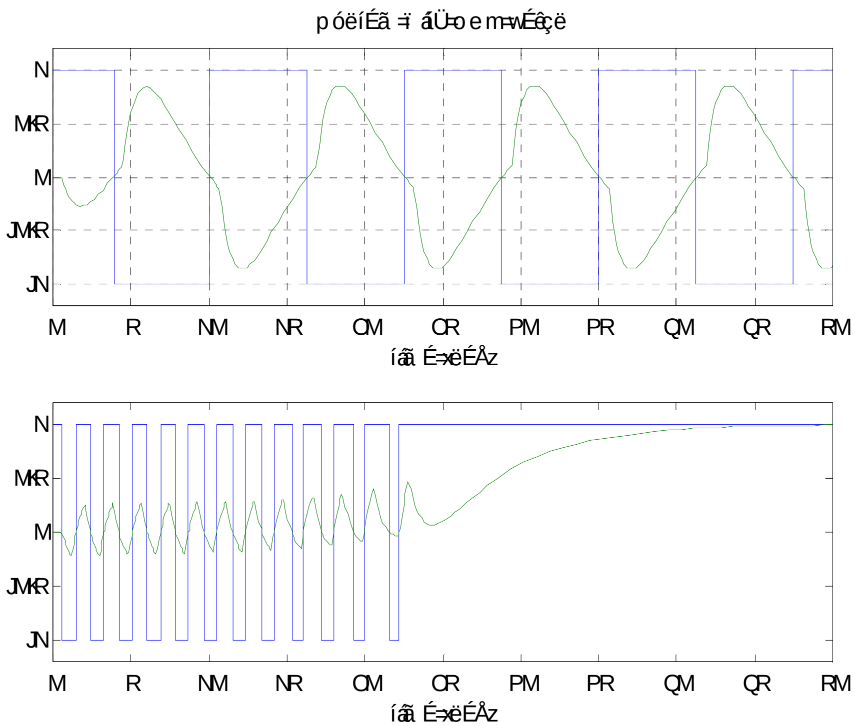
Relay with hysteresis

Ideal and Saturation Relay



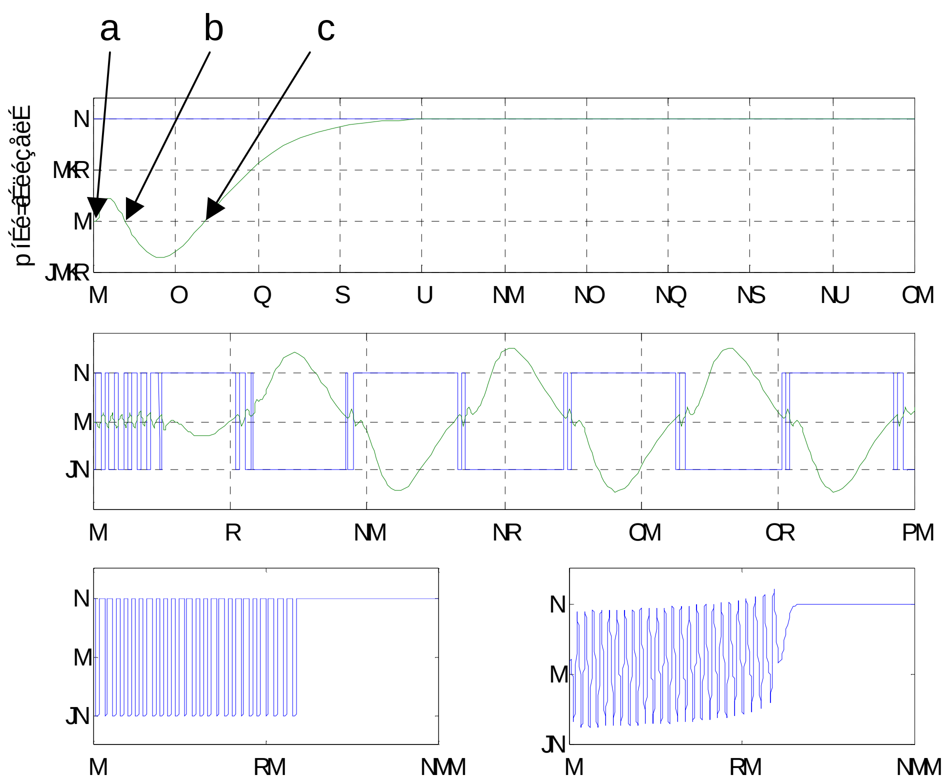
$$G(s) = \frac{12.8e^{-s}}{16.8s+1}$$

System with RHP Zeros



$$G(s) = \frac{(-3s+1)e^{-0.6s}}{(5s+1)(s+1)}$$

System with two RHP zeros



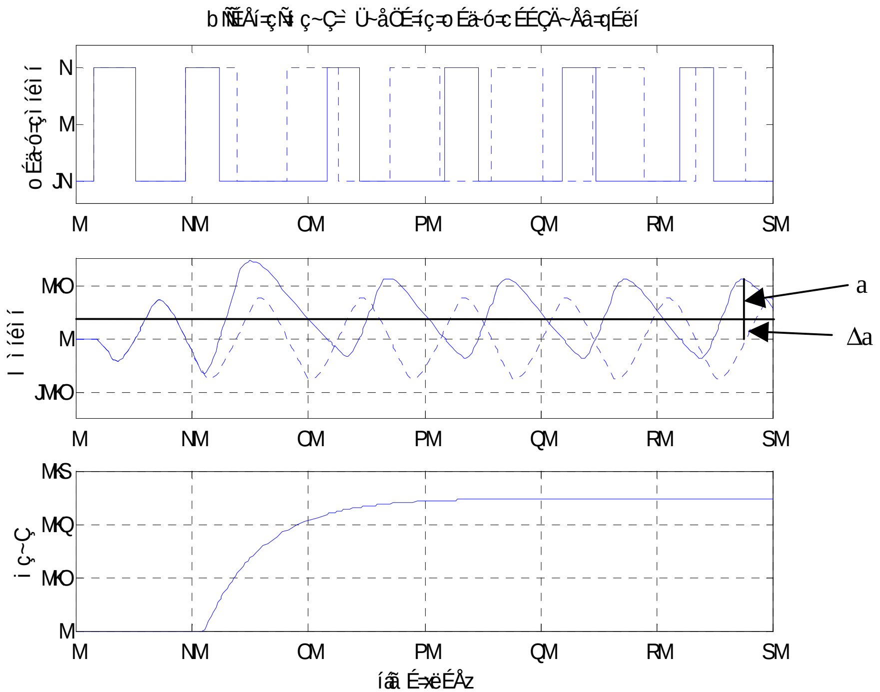
Switched at b

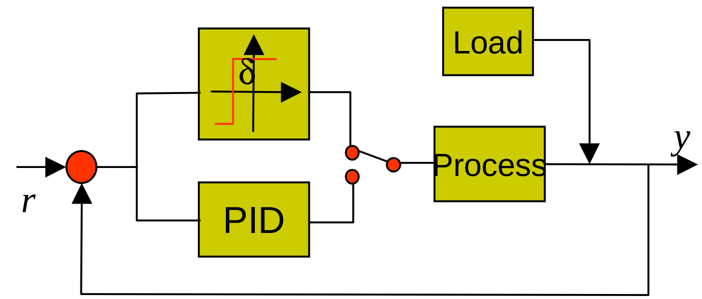
Switched at c

$$G(s) = \frac{(-s+1)^2 e^{-0.1s}}{(0.8s+1)^3}$$

Wrong sign of the system

Load Disturbance Effect



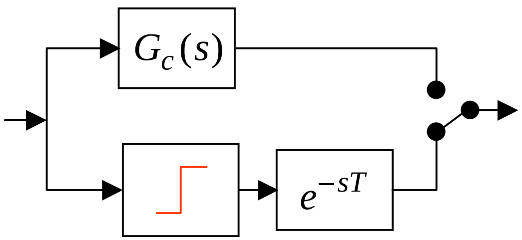


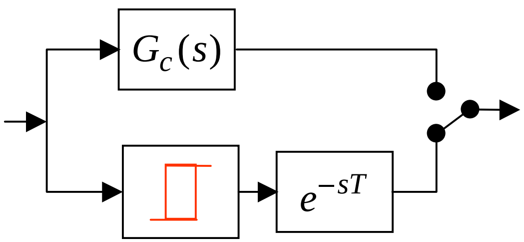
$$G(s) = \frac{e^{-1.5s}}{(10s+1)(s+1)}$$

$$G_L(s) = \frac{e^{-s}}{5s+1}$$

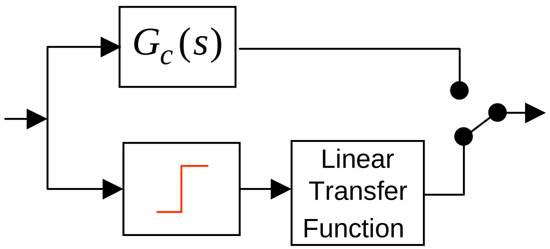
Multiple point estimation

- **Time delay element in series with a relay**





(Besancon-Voda and Roux, Buisson, 1997)



(Schei, 1992)

Tuning Using Optimization Methods

- Based on optimization of certain, mostly integral criteria
- The technique dates back to papers (Johnson, 1968; Athans, 1971; Williamson & Moore, 1971)

$$J_n(\theta) = \int_0^\infty \left[t^n e(\theta, t)\right]^2 dt$$

$$n = 0 \quad ISE$$

$$n = 1 \quad ISTE$$

$$n = 2 \quad IST^2E$$

- Most of the methods based on FOPD system
- PID tuned in frequency domain using an optimization (Liu and Dailey, 1999)
- Comparative study (Ho *et al.,* 1999)

Internal Model Control Tuning

- Developed by Morari and co-workers (Garcia and Morari, 1982)
- IMC is a general design technique - PID is a special case
- This is analytical method of PID design based on FOPD model.
- Tuning by this method considered in (Chien &Fruehauf, 1990; Rotstein & Levin, 1991; Jacob & Chidambaram, 1996).
- Comparative study between IMC based and frequency based tuning (Hang *et al.,* 1994)
- Several IMC schemes compared in (Vandeursen & Peperstraete, 1996)
- IMC has very good robustness (Scali *et al.,* 1992)
- Simplified tuning rules for IMC presented in (Fruehauf *et al.,* 1994)
- Improved filter design for IMC proposed in (Horn, 1996).

Other Tuning Methods

- Approximation of pure time delay by Pade approximation (Yutawa & Seborg, 1982) of FOPD model to get second order system.
- Iterative technique to solve transcendental equation (Lee, 1989)
- Pattern recognition based adaptive controller (Cao & McAvoy, 1990)
- Transient response of second order plus time delay (Hwang, 1995)
- Graphical tuning based on the parametric D-stability partitioning (Shafei & Shenton, 1994)
- Gain scheduling tuning (McMillan *et al.,* 1994)
- Tuning based on the closed-loop system specification (Abbas, 1994)
- Delay compensation PID tuning formula based on Smith predictor (Tsang *et al.,* 1994)
- Pole-placement method (Hwang & Shiu, 1994)
- Model-based PID tuning (Huang *et al.,* 1996)
- Kessler;s Symmetric optimum principle (Voda & Landau, 1995)

Kessler's Symmetrical Optimum Principle

- Based on two Kessler's papers from fifties which describe PID design technique based on Bode diagrams
- The idea is based on the idea that the plant transfer function be as close as possible to one at low frequency by accommodating $G(0)=0$ and $d^iG(j\omega)/dt^i=0$ at $\omega=0$ for i as high as possible.
- Kessler's principle says that:
  - the gain cross over frequency of the compensated system should be placed at $\omega_{cg}=1/2\tau_e$, where τ_e is equivalent time constant of all noncompensable time constants (sum of fast time constants and time delay).
  - The slope of the Bode diagram at the gain crossover frequency is minus *20 dB/dec*
  - the PID controller is chosen such that it preserves the slope of minus *20 dB/dec* for one octave to the right and *m* octaves to the left (*m* is the number of compensated time constants)

Kessler's Symmetrical Optimum Principle

$$G(s) = \frac{Ke^{-s\tau_n}}{(1+\tau_1 s)(1+\tau_2 s)\ldots(1+\tau_{n-1} s)(1+\tau_n)} \approx G_{app}(s) = \frac{K}{(1+\tau_1 s)(1+\tau_2 s)(1+\tau_e s)}, \ \omega \le \frac{1}{\tau_e}$$

For m=2 and $\tau_1 \ge \tau_2 >> (\tau_3 + \tau_4 + \ldots + \tau_n) = \tau_e$

In the neighborhood of the gain crossover frequency, $\omega_{cg} = 1/2\tau_e$, *G(s)* is approximated by

$$G(s) = \frac{K}{(1+\tau_1 s)(1+\tau_e s)}, \text{ with } \tau_1 \ge 4\tau_e$$

Tuning of PID controller by Kessler's method (Voda and Landau, 1995)

Controller Type	Assumed Model	Controller Parameters
PI	$G_1(s) = \frac{K}{(1+\tau_1 s)(1+\tau_e s)}, \tau_1 \ge \tau_e$	$K_p = \frac{0.5\tau_1}{K\tau_e}, \quad T_i = 4\tau_e$
PID	$G_2(s) = \frac{K}{(1+\tau_1 s)(1+\tau_2 s)(1+\tau_e s)}$, $\tau_1, \tau_2 \ge \tau_e$	$T_d = \frac{4\tau_2\tau_e}{\tau_2 + 4\tau_e}$, $T_i = \tau_2 + 4\tau_e$, $K_p = \frac{\tau_1(\tau_2 + 4\tau_e)}{8K\tau_e^2}$

Kessler's method salient features

- Produces good phase and gain margins by imposing the slope *20 dB/dec* around the gain crossover frequency
- Handles well nonlinearities and time varying parameters, and takes into account unmodeled dynamics (represented by the equivalent time constant τ_e (Voda and Landau, 1995)
- The frequency 1/ τ_e can be found from the Nyquist diagram where the phase margin is around 45°. This frequency also represents the closed loop bandwidth
- This frequency can be determined from a relay with hysteresis feedback experiment, as follows:

PID Tuning by Kessler-Landau-Voda method (KLV)

Controller Type	Assumption	Controller Parameters
PI	$\omega_{135} = \frac{\alpha}{\tau_e}$	$K_p = \frac{1}{3.5\tau_e}, \quad T_i = \frac{4\alpha}{\omega_{135}} = \frac{4.6}{\omega_{135}}$
PID	$\tau_2 \approx 1/\omega_{135} \Rightarrow \omega_{135} = 1/\beta\tau_e, 1 < \beta < 2$	$K_p = \frac{\beta(4+\beta)}{8\sqrt{2}G(\omega_{135})}$, $T_i = \frac{4+\beta}{\beta\omega_{135}}, T_d = \frac{4}{(4+\beta)\omega_{135}}$

Industrial Controllers

- ABB Commander 355:
  - Gain scheduling, feedforward, cascade, ratio control, autotune for 1/2 wave od minimal overshoot
- Foxboro 762C:
  - Exact Self-tuning control, dynamic compensation: lead/lag, impulse, dead time.
- Fuji Electric PYX:
  - Autotuning, fuzzy logic feedback control
- Honeywell (few different models):
  - Self-tuning, autotuning, gain scheduling, fuzzy logic overshoot suppression
- Yokogawa:
  - Autotuning, overshoot suppression (at sudden change of setpoint), gain scheduling

Implementation Issues

- Commercial controllers
  - Of the shelf units
  - Mostly digital versions with sophisticated auto-tuning features
  - Used in SISO (or multi-loop control architectures)
  - Give satisfactory results (according to the Corning engineers)
  - Digital controllers have 0.1s sampling period - good for process control
  - Contain many of additional features based on many years of application experience (integral windup prevention, integral preload, derivative limiting, bumpless transfer
  - The above features make PID safe to use.

Embedded Controllers

- Customized to the specific needs (when there are special requirements for speed, size, ...)
- Needed when the custom version of PID control (combined with monitoring, alarm processing, communication software … if needed)
- Can accommodate virtually any tuning method
- Very fast control loops require fixed-point arithmetic and special electronics for implementation (DSP, FPGA,…)
- Example: optical amplifier gain and output optical power control - needs very fast sampling rates.

Conclusions

- PID (PD, PI) controllers received lot of attention during ‘90
- Centennial work of Ziegler and Nichols (1942) still widely used in industrial applications and as a enchmark for new techniques
- Despite of a huge number of theoretical and application papers on tuning techniques of PID controllers, this area still remains open for further research
- There is lack of comparative analysis between different tuning techniques
- No common benchmark examples
- There is a number of industrial controllers based on modern tuning techniques
- Embedded controllers are good candidates for new PID techniques
- The area is still open for research