Linear Versus Circular Convolution

How do we avoid illegal terms (owing to circular nature of periodic signals) showing up in circular convolution? The remedy is to pack the sequences with enough zeros. A close comparison of linear and circular convolutions shows that by packing $N$ zeros at the tail end of $N$ point sequences and performing a $2N$ point circular convolution yields the same results as linear convolution of $N$ point sequences.

The following figure illustrates for $N = 4$ both linear and circular convolutions. For linear convolution, the values represented by $\Delta$s are zeros, however for circular convolution the values represented by $\Delta$s are not necessarily zero since the given sequences are periodic with period $8$. As seen from the figure, in the convolutional operation, the $\Delta$ at any location gets multiplied by a zero and thus has no effect whatsoever in the eventual result. As such linear convolution of finite sequences of length $4$ and circular convolution of periodic sequences of length $8$ with $4$ zeros at the tail end of each period yield the same results.

The outputs $y[n] = x[n] * h[n]$ and $y_c[n] = x[n] \circ h[n]$ as given by the above figure are

\[
\begin{align*}
    y[0] &= y_c[0] &= x[0]h[0] \\
\end{align*}
\]