Solutions for Midterm Exam 1

1. Three major steps in the PCM process, as per Figure 2.2: (1) Sampling, (2) Quantizing and (3) Encoding. For voice, the audio signal is sampled at 8000 times per second. Each sample is quantized to one of a finite number of amplitude levels that is encoded with 8 bits → \((8000 \text{ samples/second}) \times (8 \text{ bits/sample}) = 64000 \text{ bps})

2. Information message 100, \(g(x) = x^2 + 1\). What is the encoded channel word?
   a) Even parity: \(100 \rightarrow 1001\) (even number of ones in transmitted channel word)
   b) CRC: \(100 \rightarrow 10001\) (obtained via long division or shift register implementation)

List the received word sequences with undetectable word sequences?
   c) Even parity: Those received word sequences with an even number of bit errors (0101, 0011, 0000, 1111, 1100, 1010 and 0110).
   d) CRC: Those received word sequences whose received error vector has 101 as a factor (i.e., \(e \in \{10100, 01010, 00101, 10001, 11110, 11011, 01111\}\)). Equivalently, those received word sequences that correspond to one of the other 7 valid code words (00000, 00101, 01010, 01111, 10100, 11011 and 11110).

3. M/M/m/m queue with \(\lambda_n = \lambda\), \(n = 0,1,...,m-1\) and \(\mu_n = m \cdot \mu\), \(n = 1,2,...,m\).
   a) State transition diagram:

   ![State Transition Diagram](image)

   b) \(\lambda \cdot p_{n-1} = n \cdot \mu \cdot p_n \Rightarrow p_n = \frac{\lambda}{n \cdot \mu} \cdot p_{n-1} \Rightarrow p_n = \left(\frac{\lambda}{\mu}\right)^n \cdot \frac{1}{n!} \cdot p_0\)

   Solving for \(p_0\): \(1 = \sum_{k=0}^{m} p_k \Rightarrow 1 = \sum_{k=0}^{m} \left(\frac{\lambda}{\mu}\right)^k \cdot \frac{1}{k!} \cdot p_0 \Rightarrow 1 = p_0 \cdot \sum_{k=0}^{m} \left(\frac{\lambda}{\mu}\right)^k \cdot \frac{1}{k!}\)

   \(\Rightarrow p_0 = \left(\sum_{k=0}^{m} \left(\frac{\lambda}{\mu}\right)^k \cdot \frac{1}{k!}\right)^{-1}\)

   Substituting result for \(p_0\) into expression for \(p_n\): \(p_n = \frac{\left(\frac{\lambda}{\mu}\right)^n \cdot \frac{1}{n!}}{\sum_{k=0}^{m} \left(\frac{\lambda}{\mu}\right)^k \cdot \frac{1}{k!}}\)

   c) \(p_n = p_m = \frac{\left(\frac{\lambda}{\mu}\right)^m \cdot \frac{1}{m!}}{\sum_{k=0}^{m} \left(\frac{\lambda}{\mu}\right)^k \cdot \frac{1}{k!}}\)
4. M/G/1 queue with deterministic service times (i.e., M/D/1 queue). $\lambda_1 = 1, \lambda_2 = 2, \mu_1 = 10$ and $\mu_2 = 5$.

   a) \[ \rho = \frac{\lambda_1}{\mu_1} + \frac{\lambda_2}{\mu_2} = \frac{1}{10} + \frac{2}{5} = \frac{1}{2}. \] (Aside: $\rho_1 = \frac{\lambda_1}{\mu_1} = \frac{1}{10}$ and $\rho_2 = \frac{\lambda_2}{\mu_2} = \frac{2}{5}$)

   b) Non-preemptive priority service:
   
   Deterministic service rates imply
   
   $\overline{X}_1 = \frac{1}{\mu_1} = \frac{1}{10} \Rightarrow \overline{X}_1^2 = (\overline{X}_1)^2 = \frac{1}{100}$ and $\overline{X}_2 = \frac{1}{\mu_2} = \frac{1}{5} \Rightarrow \overline{X}_2^2 = (\overline{X}_2)^2 = \frac{1}{25}$

   M/G/1 queue with $n$ priority classes: $\overline{W}_k = \frac{1}{2} \cdot \left(1 - \sum_{j=1}^{k-1} \rho_j \right) \cdot \left(1 - \sum_{j=1}^{k} \rho_j \right)$

   $\Rightarrow \overline{W}_2 = \frac{\lambda_1 \cdot \overline{X}_1^2 + \lambda_2 \cdot \overline{X}_2^2}{2 \cdot (1 - \rho_1) \cdot (1 - \rho_1 - \rho_2)} = \frac{(1) \cdot (1/100) + (2) \cdot (1/25)}{2 \cdot (1 - (1/10)) \cdot (1 - (1/10) - (2/5))} = \frac{9/100}{(2) \cdot (9/10) \cdot (1/2)}$

   $\Rightarrow \overline{W}_2 = \frac{1}{10}$

   c) Non-priority service:

   $\lambda = \lambda_1 + \lambda_2 = 1 + 2 = 3$

   $\overline{X}^2 = \frac{\lambda_1 \cdot \overline{X}_1^2 + \lambda_2 \cdot \overline{X}_2^2}{\lambda} = \frac{(1) \cdot (1/100) + (2) \cdot (1/25)}{3} = \frac{3}{100}$

   $\overline{W}_2 = \overline{W}_1 = \overline{W} = \frac{\lambda \cdot \overline{X}^2}{2 \cdot (1 - \rho)} = \frac{(3) \cdot (3/100)}{2 \cdot (1 - (1/2))} = \frac{9}{100}$

5. $W = 4 \rightarrow s_{\text{MIN}} = 4$. An example of why $s_{\text{MIN}} - 1 = 3$ is insufficient is shown below where lost acknowledgements result in retransmission of a window of frames that are erroneously accepted as new frames. On the other hand, $s_{\text{MIN}} = 4$ is sufficient for correct operation as retransmission of frames 0-3 would be detected as duplicates at the RX because the next frame expected is frame 4.

![Diagram showing retransmission and detection of duplicates](image-url)