6 Problems, 100 Points, 80 Minutes  
Must show related work and/or provide explanation for all answers

1. **Ohm’s Law, KCL, KVL and Power.** Consider the partially solved circuit given below. Apply Ohm’s Law, KCL, KVL and power analysis appropriately to compute the following:
   
a) (3 points) Current $i_1$
   b) (3 points) Voltage $v$
   c) (3 points) Current $i_2$
   d) (3 points) Current $i_3$
   e) (3 points) Power associated with the 100V source
   f) (3 points) Power associated with the 50V source

For parts (e) and (f), clearly indicate whether the sources are generating or absorbing power.

**NOTE:** The circuit as specified is **not a valid circuit.** A consequence of this is that there is no single solution to the problem as formulated. As a result, depending on the techniques applied to solve for the requested currents and voltage, different answers may be derived. Therefore, in grading this problem, each offered solution received credit provided the analysis was logical and valid.

**An accepted solution (One of several accepted):**

a) KCL at node c: $i_1 + 9 - 5 = 0 \Rightarrow i_1 = -4 \, \text{A}$

b) KVL around loop (f, 100 V, a, 4 Ω, b, 12 Ω, c, 2 Ω, f) 
   
   $-100 + v + (12)(5) + (2)(-4) = 0 \Rightarrow v = 48 \, \text{V}$

c) Ohm’s Law at 4 Ω resistor: $i_2 = \frac{v}{4} = 12 \, \text{A}$

d) KCL at node f: $-i_1 + i_2 + i_3 = 0 \Rightarrow -(-4) + 12 + i_3 = 0 \Rightarrow i_3 = -16 \, \text{A}$

e) $p_{100V} = 100i_2 = (100)(12) = 1200 \, \text{W (generating)}$

f) $p_{50V} = -100i_3 = -(50)(-16) = 800 \, \text{W (generating)}$
2. *(12 points) Voltmeter Design.* Consider the *d’Arsonval voltmeter* shown in the figure below. Solve for the value of $R_V$ to yield a 50V full-scale reading across the terminals a,b. (1 mA = $10^{-3}$ A, 100 mV = 0.1 V)

Solution:

\[ V_{FS} = 100 \text{ mV}, \quad I_{FS} = 1 \text{ mA} \]

\[ v_{FS} = 50 \text{ V} = I_{FS} R_V + V_{FS} \]

\[ 50 \text{ V} = \left(10^{-3}\right) R_V + \left(100 \cdot 10^{-3}\right) \]

\[ R_V = \frac{50 - 0.1}{10^{-3}} = 49.9 \text{ k}\Omega \]
3. (20 points) Δ-to-Y transformation. Applying a Δ-to-Y transformation to the mesh formed by the 10Ω, 10Ω and 5Ω resistor elements, compute the equivalent resistance seen across the terminals of the voltage source in the figure below.

Solution:

Next, series and parallel resistor transformations can be applied to compute \( R_{eq} \):

\[ R_{eq} = 17 \, \Omega \]
4. **(20 points) Node-Voltage Method.** Apply the node-voltage method to compute the power associated with the dependent voltage source (clearly indicate whether this source is generating or absorbing power).

Solution:

Choose node 5 as the reference node.

Having chosen node 5 as the reference node, the following constraint equations may be obtained:

\[ v_4 = -10 \text{ V} \]
\[ v_3 = 2 \text{ V} \]
\[ i_x = -\frac{v_1}{2} \]
\[ v_2 = v_1 + 4i_x = v_1 + 4\left(-\frac{v_1}{2}\right) = -v_1 \]

KCL at supernode (1,2):

\[ \frac{v_1}{2} + \frac{v_1 - v_3}{1} + \frac{v_2 - v_3}{2} = 0 \]
\[ \Rightarrow \frac{v_1}{2} + \frac{v_1 - 2}{1} + \frac{-v_1 - 2}{2} = 0 \Rightarrow v_1 = 3 \text{ V} \Rightarrow v_2 = -3 \text{ V} \Rightarrow i_x = -\frac{3}{2} \text{ A} \]

Current in branch of dependent voltage source (going from node 1 to node 2):

\[ i_{dep} = \frac{v_2 - v_3}{2} = \frac{-3 - 2}{2} = -\frac{5}{2} \text{ A} \]

Power due to dependent voltage source:

\[ \Rightarrow P_{dep} = (4i_x)i_{dep} = (4)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right) = 15 \text{ W (delivered)} \]
5. **(15 points) Mesh-Current Method.** Consider the following circuit with 3 unknown mesh currents \(i_1, i_2\) and unknown current \(i_x\) in the 20\(\Omega\) branch. Write the independent equations needed to solve for the unknowns (it is not necessary to solve the equations).

\[2\text{ constraint equations:}\]

Net current in 20\(\Omega\) branch: \(i_x = i_2 - i_1\)

3 A source: \(i_3 = 3 + i_2\)

\[2\text{ mesh equations:}\]

KVL around Mesh 1: 
\[-5i_x + 10i_1 + 40(i_1 - i_3) + 20(i_1 - i_2) = 0\]

KVL around supermesh (2,3): 
\[10i_x + 20(i_2 - i_1) + 40(i_3 - i_1) + 50i_3 = 0\]
6. (15 points) **Source Transformations.** In the circuit shown below, determine the current $i_0$ using source transformations. (Hint for source transformation formula: $v_s = i_s R$)

![Circuit Diagram](image)

**Solution:**

Source transformation of 30 V source:

![Parallel Equivalent](image)

Source transformation of 6 A source:
Source transformation of 3 A source:

\[
\begin{align*}
\text{Adding series voltage sources:} \\
\Ohm's \ Law: \quad i_0 &= \frac{18 \text{ V}}{9 \Omega} = 2 \text{ A}
\end{align*}
\]