Make sure that there are thirteen pages in this exam excluding this cover page.

This is a closed-book exam. Except calculators, no other material is permitted. Do all your work on these sheets. If more space is required, work on the back of these sheets and indicate accordingly.

Note that no credit will be given if proper explanation is lacking.

First five problems should take considerably less time than the last four problems because of complex algebra.

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Total maximum points = 100
Total points earned by the student =
Problem 1 (11 points):
Consider the circuit shown. What is the relationship between $R$ and $R_L$ so that the equivalent resistance seen by the source $V_1$ at the terminals $ab$ equals $R_L$? Justify your answer.
Hint: Answer is one of the following, $R_L = R, R_L = 2R, R_L = 3R, R_L = 4R$.

![Circuit Diagram]

The equivalent resistance at the terminals $ab = \text{Re}ov$

\[
\frac{(R+R_L)4R + R}{4R+R+R_L} = R_L
\]

We would like to have $\text{Re}ov = R_L$

\[
\frac{(R+R_L)4R}{5R+R_L} + R = R_L
\]

\[
(4R+R_L)(4R)+(5R+R_L)R = (5R+R_L)R_L
\]

\[
4R^2 + 4R^2 + 5R^2 + 5RR_L = 5RR_L + R_L^2
\]

\[
R_L = \frac{3R}{5}
\]

Consider the circuit shown.
Calculate the ratio $\frac{V_2}{V_1}$.
Hint: Answer is one of the following, $\frac{V_2}{V_1} = 0.25, \frac{V_2}{V_1} = 0.5, \frac{V_2}{V_1} = 0.75, \frac{V_2}{V_1} = 1$.

![Circuit Diagram]

We note that in this circuit

$R = 1\Omega, R_L = 3\Omega$.

**The equivalent resistance seen by $V_1$ is $3\Omega$.

** $I_1 = \frac{V_1}{3}$ mA

Now using current division rule,

\[
I_2 = \frac{V_1}{3} \frac{4+4}{4+4} = \frac{V_1}{6} \text{ mA}
\]

\[
V_2 = \frac{V_1}{6} \text{ mA} 3\Omega = \frac{V_1}{2}
\]

\[
\frac{V_1}{V_2} = 2 \quad \frac{V_2}{V_1} = 0.5
\]
Problem 2 (11 points):
In the circuit shown, calculate the voltage $V_2$ by using any method.

We will do this by nodal analysis.

Mark the node voltages as shown.
Note that $V_1 = 12\, V$, $V_3 = -6\, V$.

Write a node equation at the central node

$$\frac{V_2 - 12}{12\, \Omega} + \frac{V_2 - (-6)}{12\, \Omega} + \frac{V_2}{6\, \Omega} = 0.$$ 

$$V_2 - 12 + V_2 + 6 + 2V_2 = 0$$

$$4V_2 = 6$$

$$V_2 = \frac{6}{4} = \frac{3}{2} \, V.$$
Problem 3 (11 points):
In the circuit shown, calculate the current $I_0$ by using any method.

We will do this by using current method. Select $I_1$, $I_2$, and $I_3$ as shown.

$I_1 = 2 \text{mA}$ and $I_2 + I_3 = 4 \text{mA}$ by direct observation.

Writing the super loop equation $ABCBEFA$, we get

$$+2k(I_1 + I_2) + (I_1 + I_3)1k + 6 + I_3 1k - I_2 2k = 0$$

$$\therefore +3I_1k - 4I_2k + 2I_3k = -6$$

Substituting $I_1 = 2 \text{mA}$ and $I_3 = 4 \text{mA} - I_2$, we get

$$6 - 4I_2k + 2[4 \text{mA} - I_2]k = -6$$

$$-6I_2k = -26$$

$$\therefore I_2 = \frac{26}{6} = \frac{13}{3} \text{mA}$$

$I_0 = I_1 - I_2 = 2 - \frac{13}{3} = -\frac{1}{3} \text{mA}$
Problem 4a (5 points):
The Op-Amp in the circuit shown is ideal. Calculate \( \frac{V_0}{V_i} \) by using any method.

Let the voltage at both the non-inverting as well as inverting terminals be \( V_1 \) with respect to the ground. Then we have the following node equations:

\[
\begin{align*}
\frac{V_1 - V_i}{1k} + \frac{V_1 - V_0}{2k} &= 0 \\
\frac{V_1 - V_0}{3k} + \frac{V_1}{1k} &= 0.
\end{align*}
\]

Solve for \( V_1 \):

\[ V_1 = \frac{V_0}{4} \]

\[
\begin{align*}
3V_1 - 2V_i - V_0 &= 0 \\
\frac{3}{2}V_0 - 2V_i - V_0 &= 0 \\
2V_i - \frac{1}{4}V_0 &= 0.
\end{align*}
\]

\[ V_0 = -8V_i \]

\[ \frac{V_0}{V_i} = -8 \]
Problem 4b (6 points):
The Op-Amps in the circuit shown are ideal. Calculate $V_{o2}$ by using any method.

We use node voltage method.
Mark the node voltages as shown.

Write the node equations as:

\[
\frac{15 - V_{o1}}{0.5k} + \frac{15 - V_{o2}}{2k} + \frac{15 - 10}{5k} = 0
\]

\[
\frac{10 - V_{o2}}{0.4k} + \frac{10 + 10 - 15}{1k} + \frac{10 - 15}{5k} = 0
\]

\[
25 - 25V_{o2} + 10 + 2 - 3 = 0
\]

\[
25V_{o2} = 34
\]

\[
V_{o2} = \frac{34}{25} = 1.36 \text{ V}
\]
Problem 5 (11 points):
In the circuit shown, calculate $V_0$
by using superposition method.
Hint: The circuit is partially duplicated
two times on this page and one time
on the next page just for convenience.

We first solve the circuit
using only 12 V source.
This means we need to
open the 2 mA current
source.

Then we have

\[
\begin{align*}
6k & \quad 3k & \quad 6k \\
\hline
2k & \quad 3k & \quad 2k \\
\end{align*}
\]

\[\Rightarrow 2k = \begin{align*}
3k & \quad 2k \\
\end{align*}\]

\[
\text{Current I } = 2\text{ mA}
\]

Then by inspection, current through 4 k ohm is 1 mA, giving

\[
V_0 \text{ due to } 12\text{ V source } = -4\text{ V}.
\]

Continued on the next page
We will now solve the circuit using only 2 mA source.
This means we need to short the 12 V source.

By inspection, the currents are as marked.
Thus the current through 4 kΩ is 0.5 mA going down to
V₀ due to 2 mA source is 2 V.

The overall voltage due to both sources is
V₀ = -4 + 2 = -2 V

Check: By converting the 12 V source in series with 3 kΩ
into a current source in parallel with 3 kΩ, we can
equivalently draw the network as

V₀ = -2 V
Problem 6 (15 points):

Find the impedance $Z_{ab}$ in the circuit seen in this figure. Express $Z_{ab}$ in both polar and rectangular form.

\[Z_1 = 10 - j40 \Omega\]

\[Z_2 = (5 - j10)(10 + j30) = \frac{10 - j10 \Omega}{15 + j20}\]

\[Z_3 = \frac{20(20)}{20 + j20} = 10 + j10 \Omega\]

\[\therefore Z_{ab} = Z_1 + Z_2 + Z_3 = 30 - j40 \Omega = 50/\angle -53.15^\circ \Omega\]
Problem 7 (11 points):

Find the Norton equivalent circuit with respect to the terminals a, b for the circuit in this figure.

Hint: Find the short circuit current through the terminals a and b. Next find the impedance seen from the terminals a and b when the source is open.

The Norton current $I_N$ equals the short circuit current as shown. By current division rule:

$$I_N = \frac{(16A)(25)}{25 + 15 + j30} = 6.4 - j4.8 \, A$$

Impedance seen from the terminals a and b when the 16A source is open.

$$Z_{a/b} = Z_N = \frac{(-150)(40 + j30)}{40 + j30 - j30} = 50 - j25 \, \Omega$$

Thus the Norton equivalent circuit is as shown:
Problem 8 (11 points):

Three loads are connected in parallel across a 2400 V (rms) line, as shown in this figure. Load 1 absorbs 18 kW and 24 kVAR. Load 2 absorbs 60 kVA at 0.6 pf lead. Load 3 absorbs 18 kW at unity power factor.

(a) Calculate the total complex power of all the three loads together.

\[
\begin{align*}
\text{Load 1} & : S_1 = 18 + j 24 \text{ kVA} \\
\text{Load 2} & : |S_2| = 60 \text{ kVA} \\
& = 60 \cos \theta - j 60 \sin \theta \text{ kVA} \\
& = 36 - j 48 \text{ kVA} \\
\text{Load 3} & : S_3 = 18 \text{ kW} \\
\end{align*}
\]

\[S_1 = S_1 + S_2 + S_3 = 72 - j 24 \text{kVA}\]

(b) Calculate the current supplied by the 2400 V source.

\[2400I^* = (72 - j 24) \times 10^3; \quad \therefore I = 30 + j 10 \text{ A}\]

This problem is continued on the next page.
(c) What is the equivalent impedance of all the three loads together?

\[
Z = \frac{2400}{30 + j10} = 72 - j24 \Omega = 75.89 \angle -18.43^\circ \Omega
\]

(d) Find the power factor as seen by the 2400 V source.

\[
\text{pf} = \cos(-18.43^\circ) = 0.9487 \text{ leading}
\]
Problem 9 (11 points):
A block-diagram of a circuit is shown. The phasor voltage $V_{ab}$ is $480 + j60$ V RMS when there is no external load connected at the terminals a and b. However, when an impedance of $100 + j60 \Omega$ is connected across a and b, the value of $V_{ab}$ is $240 - j80$ V RMS.

(a) Draw the Thevenin equivalent circuit of the circuit in the block-diagram. Identify the Thevenin voltage and the Thevenin impedance.

$$V_{Th} = \text{Open circuit voltage} = 480 \text{ V}$$

From the given data, we have

Write a node equation at 'a' to get

$$\frac{(240-j80)(-480)}{100} + \frac{240-j80}{100} = 0$$

$$\therefore Z_{Th} = \frac{-100(240+j80)}{-240-j80} = 80 + j60 \Omega$$

This problem is continued on the next page.
(b) Find the impedance that should be connected across a and b so that the maximum (real) power can be transferred to it.

The load $Z_L$ is a conjugate of $Z_{TN}$ for maximum power transfer to $Z_L$.

$\therefore Z_L = 80 - j60\Omega$

(c) Find the maximum (real) power that can be transferred to the load impedance.

$I = \frac{480\angle 0^\circ}{160\angle 0^\circ} = 3\angle 0^\circ$ A (rms)

$P = (9)(80) = 720\text{W}$