Principles of Electrical Engineering - I (14:332:221)
Chapter 5 Notes

Op Amp Terminals (1/2)

- Primary Terminals of interest:
  - Inverting input
  - Noninverting input
  - Output
  - Positive power supply ($V^+$)
  - Negative power supply ($V^-$)

E.g., 8-lead dual-in-line package (DIP)

Op Amp Terminals (2/2)

Terminal Voltages and Currents (1/5)

- All voltages measured with respect to a common ground

Terminal Voltages and Currents (2/5)

- Current reference directions are into the op amp terminals

Terminal Voltages and Currents (3/5)

- Op amp is characterized as a linear amplifying element with gain $A$
  - Linear region of operation depends on the values of $V^+$ and $V^-$
  - Input-output relation depends on the difference between the two input voltages, $v_p$ and $v_n$

\[
V_v = \begin{cases} 
  -V_{CC} & \text{if } v_p - v_n < -V_{CC} \\
  -V_p & \text{if } -V_p \leq v_p - v_n \leq +V_p \\
  +V_{CC} & \text{if } +V_p - v_n \leq +V_{CC} 
\end{cases}
\]
Terminal Voltages and Currents (4/5)

• Gain $A$ is typically $>10^4$ ⇒
  - $|v_p - v_n| < 2 \text{ mV}$
  - In ideal op amp, $A = \infty$, yielding a “virtual short” condition at the op amp input:
    
    $$v_{ip} = v_a$$
    
    5.2
  - Virtual short condition is maintained via negative feedback
    - Signal from output terminal is fed back to the inverting terminal
    - Lack of negative feedback typically results in saturation
  - Ideal op amp presents infinite resistance at input terminals (typically 1 MΩ or more):
    
    $$i_{in} = i = 0$$
    
    5.3

Terminal Voltages and Currents (5/5)

• KCL at the op amp yields:
  
  $$i_{in} = i = 0$$
  
  5.4

• Combining (5.3) and (5.4):
  
  $$v_p = -v_n$$
  
  5.5

Inverting Amplifier (1/2)

• Inverting Amplifier: Output voltage is an inverted scaled replica of the input

  $$i_i + i_f = i_a$$
  
  5.6

Since $v_p = 0$, $v_n = 0$ by (5.2):

  $$i = \frac{v}{R}$$
  
  5.7

$$i = \frac{v}{R}$$

  5.8

Recalling (5.3), $i = 0$:

  $$v = -\frac{R}{R}v$$
  
  5.10

Inverting amplifier operating in open loop:

Since for ideal op amp, $i = 0$ (and $i = 0$, in practice):

  $$\Rightarrow v = v$$
  
  5.12

If there is no feedback path, $A$ is known as the open loop gain

Summing Amplifier (1/2)

\[ \frac{v_v - v}{R} + \frac{v_v - v}{R} + \frac{v_v - v}{R} + i = 0 \]

5.13

\[ v = \left(\frac{R}{R}v + \frac{R}{R}v + \frac{R}{R}v\right) \]

5.14

Summing Amplifier (2/2)

\[ v = \frac{R}{R}(v + v + v) \]

5.15

And if $R = R$, $\Rightarrow v = (v + v + v)$

5.16
Non-Inverting Circuit

Because $i_+ = i_- = 0$

$\Rightarrow v_+ = v_-$

And because $v_+ = v_-$

$\Rightarrow v_+ = v_-$

By voltage division:

$v_+ = \frac{v_+ R_+}{R_+ + R}$

$\Rightarrow v_+ = \frac{R_+}{R_+ + R}$

5.18

Difference Amplifier (1/3)

Voltage output is proportional to the difference between the difference between the two input voltages ($v_+$ and $v_-$)

Difference Amplifier (2/3)

Constraints:

$i_+ = i_- = 0$

5.20

$v_+ = \frac{R_+}{R_+ + R}$

5.21

KCL at 2: $v_+ - v_+ + \frac{v_+ - v_+}{R_+} + i_+ = 0$

5.19

Difference Amplifier (3/3)

Combining (5.19), (5.20) and (5.21):

$v_+ = \frac{R_+}{R_+ + R} v_+ - \frac{R_+}{R_+ + R} v_-$

5.22

Can make voltage scaling factors (for $v_+$ and $v_-$) equal by setting:

$\frac{R_+}{R_+} = \frac{R_+}{R_+ + R}$

5.23

$\Rightarrow v_+ = \frac{R_+}{R_+} (v_+ - v_-)$

5.24

Difference Amplifier Performance (1/5)

• Differential mode input: Difference between the two input voltages:

$v_{\text{diff}} = v_+ - v_-$

5.25

• Common mode input: Average of the two input voltages:

$v_{\text{com}} = \frac{(v_+ + v_-)}{2}$

5.26

Difference Amplifier Performance (2/5)

• Substituting (5.27) and (5.28) into (5.23):

$v_+ = \frac{R_+ (R_+ + R)}{R_+ (R_+ + R)} v_+ + \frac{R_+ (R_+ + R)}{2 R_+ (R_+ + R)} v_-$

5.29

$\Rightarrow v_+ = \frac{R_+}{R_+} v_+ + \frac{R_+}{R_+} v_-$

5.30

• Substituting $R_+ = R_-$ and $R_+ = R_-$ into (5.29):

$v_+ = \frac{R_+}{R_+} v_+ + \frac{R_+}{R_+} v_-$

5.31
Difference Amplifier Performance (3/5)

• By (5.31), ideal differential amplifier has zero common mode gain and large differential mode gain

• Suppose the resistor values $R_a$, $R_b$, $R_c$, and $R_d$ do not precisely satisfy (5.23):

  $R_c = (1 - \varepsilon)R_a \
  R_d = (1 - \varepsilon)R_b \Rightarrow R_c = R_a \quad R_d = R_b \quad 5.33$

  $A_m = \frac{R_c (1 - \varepsilon)R_d - R_b R_d}{R_c + (1 - \varepsilon)R_d} = \frac{-\varepsilon R_a R_b}{R_a + (1 - \varepsilon)R_b} \quad 5.34$

  $A_m = \frac{R_c (1 - \varepsilon) R_d}{R_c + (1 - \varepsilon) R_d} = \frac{-\varepsilon R_a R_b}{R_a + (1 - \varepsilon) R_b} \quad 5.35$

  $A_m = \frac{R_c (1 - \varepsilon) R_d}{R_a + (1 - \varepsilon) R_b} \quad 5.36$

More Realistic Model (2/3)

Fig. 5.16: E.g., an inverting amplifier circuit

KCL at inverting input:

$v_o - v_i + \frac{v_i - v_o}{R_f} - v_o = 0$

KCL at output:

$v_o - v_i + \frac{v_i - v_o}{R_f} - v_o = 0$

Solving for $v_i$ in terms of $v_o$ and then $v_o$ in terms of $v_i$ yields:

$v_o = \frac{R_f}{R_i} \left( 1 + \frac{R_o}{R_i} \right) \left( 1 + \frac{R_i}{R_o} \right) + \frac{R_v}{R_f} \quad 5.48$

Difference Amplifier Performance (4/5)

• (5.36) gives the mismatch on the common gain ($A_{cm}$)

• The following yields the mismatch on the differential mode gain ($A_{dm}$):

  $A_{dm} = \frac{(1 - \varepsilon)R_c (R_c + R_b) + R_b \left[ R_b + (1 - \varepsilon)R_d \right]}{2R_c \left[ R_c + (1 - \varepsilon)R_b \right]} \quad 5.37$

  $\Rightarrow A_{dm} = \frac{R_c \left[ R_b + (1 - \varepsilon)R_d \right]}{R_c + R_b + (1 - \varepsilon)R_d} \quad 5.38$

  $\Rightarrow A_{dm} = \frac{R_c \left[ R_b + (1 - \varepsilon)R_d \right]}{R_c + R_b + (1 - \varepsilon)R_d} \quad 5.39$

• When $\varepsilon = 0$, (5.34) yields $A_{cm} = 0$ and (5.37) yields $A_{dm} = R_d/R_a$ as per ideal difference amplifier

Difference Amplifier Performance (5/5)

• Common mode rejection ratio (CMRR) is defined as the ratio of differential mode gain ($A_{dm}$) to common mode gain ($A_{cm}$):

  $CMRR = \frac{A_{dm}}{A_{cm}} \quad 5.40$

• CMRR is used to measure how nearly ideal a difference amplifier is
  - The higher the CMRR, the more nearly ideal

  $CMRR = \left| \frac{R_c \left[ R_b + (1 - \varepsilon)R_d \right]}{R_c + (1 - \varepsilon) R_d} \right| \quad 5.41$

  $\Rightarrow CMRR = \left| \frac{R_c \left[ R_b + (1 - \varepsilon)R_d \right]}{R_c + (1 - \varepsilon) R_d} \right| \quad 5.42$

  $\Rightarrow CMRR = \left| \frac{R_c \left[ R_b + (1 - \varepsilon)R_d \right]}{R_c + (1 - \varepsilon) R_d} \right| \quad 5.43$

• $\varepsilon = 0 \Rightarrow CMRR = \infty$

• Use large $R_c/R_d$ to minimize effect of $\varepsilon$

More Realistic Model (3/3)

KCL at inverting input:

$v_o - v_i + \frac{v_i - v_o}{R_f} - v_o = 0$

KCL at output:

$v_o - v_i + \frac{v_i - v_o}{R_f} - v_o = 0$

Solving for $v_i$ in terms of $v_o$ and then $v_o$ in terms of $v_i$ yields:

$v_o = \frac{R_f}{R_i} \left( 1 + \frac{R_o}{R_i} \right) \left( 1 + \frac{R_i}{R_o} \right) + \frac{R_v}{R_f} \quad 5.48$