Principles of Electrical Engineering - I (14:332:221)

Chapter 4 Notes (Part 1)

Terminology (1/4)

Fig. 4.3: A circuit illustrating nodes, branches, meshes, paths and loops

Terminology (2/4)

- **Node:** A point where two or more circuit elements join, e.g., node a
  - **Essential node:** A node where three or more circuit elements join e.g., node b
- **Path:** A trace of adjoining basic elements with no elements included more than once e.g., \((v_1, R_1, R_5, R_6)\)

Terminology (3/4)

- **Branch:** A path that connects two nodes e.g., \(R_i\)
  - **Essential branch:** A path which connects two essential nodes without passing through an essential node e.g., \((v_i, R_i)\)
- **Loop:** A path whose last node is the same as the starting node e.g., \((v_1, R_1, R_5, R_6, R_4, v_2)\)

Terminology (4/4)

- **Mesh:** A loop that does not enclose any other loops e.g., \((v_i, R_i, R_i, R_i)\)
- **Planar circuit:** A circuit that can be drawn on a a plane with no crossing branches e.g., Fig. 4.3 is a planar circuit while Fig. 4.2 is a non-planar circuit

Simultaneous Equations

- A circuit with \(b\) branches requires \(b\) independent equations to solve
  - If there are \(n\) nodes in the circuit, \(n-1\) equations may be derived from KCL
- The remaining \(b-(n-1)\) equations must be derived from KVL
- Solving a circuit with respect to \(b_e\) essential branches is easier
  - \(b_e \leq b\)
  - If there are \(n_e\) essential nodes in the circuit, \(n_e-1\) equations may be derived from KCL
- The remaining \(b_e-(n_e-1)\) equations must be derived from KVL
**Node-Voltage Method (1/6)**

- A technique to simplify analysis, i.e., reduces the number of simultaneous equations
  - Analysis is done using essential nodes
  - Given $n_e$ essential nodes, node-voltage method requires $n_e-1$ equations to describe the circuit

- One of the essential nodes is chosen as the **reference node** (usually the node that connects the most branches)
- Once reference node is selected, **node voltages** are determined
  
  A node voltage is defined as the voltage rise from the reference node to a non-reference node

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**Node-Voltage Method (2/6)**

- 4 essential nodes (b, c, e, f)
  - Select node f as the reference node
  - Must solve for \(v_b\), \(v_c\) and \(v_e\)

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**Node-Voltage Method (3/6)**

- If a voltage source is the only element on a branch connecting the reference node and a non-reference node, then this greatly simplifies the circuit analysis
  - (e.g., node c: \(v_c = v_2\))
  - Reduces circuit to only two unknowns (\(v_b\) and \(v_e\))

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**Node-Voltage Method (4/6)**

- KCL at node b: 
  \[
  \frac{v_1}{R_1} + \frac{v_2 - v_1}{R_2} + \frac{v_2 - v_1}{R_3} - I = 0
  \]
- KCL at node e: 
  \[
  \frac{v_1}{R_1} + \frac{v_2}{R_2} = 0
  \]

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**Node-Voltage Method (5/6)**

- When a voltage source is the only element between two essential nodes, these nodes can be combined to form a **supernode**

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**Node-Voltage Method (6/6)**

- KCL still holds at the supernode (2,3)
  - Formation of the supernode allows \(v_1\) to be expressed in terms of \(v_2\) → Reduces circuit to a single unknown (\(v_2\))
  
  \[
  v_3 = v_2 + 10i_p
  \]
  
  KCL at supernode (2,3):
  \[
  \frac{v_2 - v_1}{5} + \frac{v_2 - v_1}{5} + \frac{v_2 + 10i_p}{100} - 4 = 0
  \]

  where \(i_p = \frac{v_2 - v_1}{5} = \frac{v_2 - 50}{5}\)
Mesh Current Method (1/2)

- Mesh-current method allows a circuit to be solved with \( h - (n_e - 1) \) independent equations
  - \( h \) = Number of essential branches
  - \( n_e \) = Number of essential nodes
- Mesh-current method applies only to planar circuits
- A mesh current is the current that exists only in the perimeter of a mesh
effectively, a virtual current that may not actually exist in any particular circuit branch (e.g., \( i_2 \) in Fig. 4.18)
- Apply KVL around \( h - (n_e - 1) \) meshes

Supermeshes (1/2)

- When a branch contains a current source, an unknown voltage is introduced to the mesh equations
  - KVL around mesh b : \( 0 = 10i_b + 2(i_b - i_a) + 3(1 - i_a) \)

Mesh Current Method (2/2)

\[ \begin{align*}
\begin{cases}
i_1 = 2 \\
12 = 3 \\
i_2 = 0
\end{cases} \\
\begin{cases}
v_1 = i_1(R_1 + R_2) - i_2R_1 \\
v_2 = -i_2R_1 + i_3(R_2 + R_3)
\end{cases} \\
\Rightarrow \\
i_1 = i_b \\
i_2 = i_b \\
i_2 = i_b - i_a
\end{align*} \]

Supermeshes (2/2)

- To create a supermesh, remove the current source by avoiding its branch when writing mesh current equations

Node-Voltage v.s. Mesh-Current (1/4)

Which method to use depends on a number of circuit issues:
- Does one method result in fewer simultaneous equations?
- Does the circuit contain supernodes?
- Does the circuit contain supermeshes?
- Will solving some portion of the circuit give the requested solution?
- Is the circuit non-planar?

Node-Voltage v.s. Mesh-Current (2/4)

Mesh-current: Must solve 5 loop equations (plus constraint for \( i_a \))

Node-voltage: Need only 2 node equations plus 1 constraint equation for \( v_2 \):

\[ v_2 = \frac{50i_a - 50(v_3 - v_1)}{300} = \frac{v_3 - v_1}{6} \]
Node-Voltage v.s. Mesh-Current (3/4)

Node-voltage: Requires 3 node-voltage equations and 2 constraint equations (due to voltage-controlled sources)

Mesh-current: Current sources reduces problem to a single mesh equation and 3 constraint equations

Source Transformations (1/4)

- A voltage source in series with a resistor may be replaced by a current source in parallel with the same resistor, or, visa versa

\[ i_s = \frac{v}{R + R_L} \text{ and } i_c = \frac{R}{R + R_L} i_s \implies i_s = \frac{v}{R} \]

Source Transformations (3/4)

Exercise 4.19

\[ \Rightarrow v_{ab} = (30)(1.92) = 57.6 \text{ V} \Rightarrow v = 48 \text{ V} \]