NAME OF THE STUDENT: [Blank]

Last four digits of Student ID #: 5124

Section # 1 (TTB4) or # 2 (W3,F4)

Circle one

Please make sure that there are 6 pages in this booklet excluding this cover page.

This is a closed-book closed-notes exam. Do all your work on these sheets. If more space is required, ask the instructor.

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Note: Most of the problems, if not all, are directly taken from Home-work problems with perhaps numbers changed and slightly modified. My goal in doing so is to encourage you to do Home-work problems. If you simply read the solutions of Home-work problems without doing them, it would not help you.

Curriculum revision: Both the curricula of Electrical option and Computer option have been revised for class year 2006 and onwards. The details are on the ECE website.
Problem 1: This is HW problem 4.74 re-phrased.
Problem 1a: Construct the Thevenin equivalent circuit with respect to the terminals $a_1$ and $b_1$. Let the Thevenin voltage be denoted by $V_{Th-1}$ and the Thevenin resistance by $R_{Th-1}$.

When there is no load at the terminals $a_1$ and $b_1$, there is no current in $b_1$. The open-circuit voltage at the terminals $a_1$ and $b_1$ is given by (Voltage division rule):

$$V_{OC} = V_{Th-1} = 200 \frac{100}{100+25} = 160V$$

Thevenin resistance is the resistance at terminals $a_1$ and $b_1$ when all the independent sources are set to zero.

$$R_{Th-1} = 10 + \frac{(100)(25)}{100+25} = 10 + 20 = 30 \Omega$$
Problem 1b: We are given the circuit drawn on the left side. However, utilizing the results of Problem 1a, we can equivalently redraw it as the one shown on the right side. Construct the Thevenin equivalent circuit with respect to the terminals \(a\) and \(b\).

**Determination of Open-circuit Voltage**

By writing the KCL for the loop ABCBA, we get:

\[ 160 = 30i_a + 20i_b + 30i_a \]

\[ i_a \cdot i_b = 2 \]

\[ V_{Th} \text{ is the voltage across } C \text{ and } D \text{ (known as break a)} \]

\[ = 20i_a + 30i_a = 50i_a = 100 \text{ V} \]

**Determination of Short-circuit Current**

By mesh analysis,

\[ 160 = 20i_b \text{ (loop ABCBA)} \]

\[ 3i_a + 2(i_a - i_b) = 0 \text{ (loop BCDAB)} \]

\[ i_a = \frac{160}{30}, i_b = \frac{50}{3} \]

Thevenin Resistance \( = \frac{V_{Th}}{I_{Th}} = \frac{100}{30} = 3.33 \Omega \)

**Determination of Thevenin Resistance by looking for the resistance at terminals \(a\) and \(b\) when all the independent sources are set to zero:**

\[ V_0 = 20(1) + 30i_b \text{ (loop BCDAB)} \]

\[ i_b = -\frac{V_0}{30} \]

\[ V_0 = -30i_b = -30 \cdot \frac{20}{30} = -20 \text{ V} \]

\[ \text{Thevenin Resistance} = \frac{V_0}{i_b} = \frac{20}{-20} = 7.5 \Omega \]

\[ V_0 = 20 \cdot (1 + i_b) + 30i_b \text{ (loop ABCBA)} \]

\[ i_b = -\frac{V_0}{30} \]

\[ V_0 = \frac{20 \cdot (30)}{80} = 7.5 \text{ V} \]

Thevenin Resistance \( = \frac{V_0}{i_b} = \frac{20}{20} = 7.5 \Omega \)
Problem 2: This is HW problem 5.33 re-phrased. Assuming that the Op-Amps are ideal, determine $v_{o1}$ and $v_{o2}$.

As usual we take the ground as the reference node.

$V_1 = \text{Voltage at A} = 3V$ (Input terminals of Top Op-Amp are Virtual Short).

$V_2 = \text{Voltage at B} = 2V$ (Input terminals of Bottom Op-Amp are Virtual Short).

**Node Equation at A:**

\[
\frac{3}{1k} - \frac{v_{o1}}{1k} + \frac{3 - V_2}{1k} = 0
\]

\[\Rightarrow 7 - v_{o1} - V_2 = 0\]

**Node Equation at B:**

\[
\frac{2}{2k} - \frac{v_{o1}}{1k} + \frac{2 - V_2}{1k} = 0
\]

\[\Rightarrow 2 - v_{o1} - V_2 = 2V\]

\[\Rightarrow v_{o2} = 2V\]

\[\Rightarrow v_{o1} = 5V\]
Problem 3: This is HW problem 5.43
re-phrased. The Op-Amp in the given circuit is
non-ideal with its input resistance $R_{in} = 500$ kΩ,
its output resistance $R_o = 0.5$ kΩ, and its gain
$A = 10^5$. Also, let $R_F = 50$ kΩ, and $R_L = 50$ kΩ.
Determine the output resistance $R_{out}$
of the circuit. An answer of $R_{out} = 0$ is not
acceptable. At least one significant bit of non-zero
value of $R_{out}$ should be indicated.

Be careful with your algebra.
A term can be neglected only if it is small
compared to another term which is added
or subtracted from it.

By using a model for Op-Amp, the output resistance
$R_{out}$ is given by the ratio $\frac{V_o}{I_o}$ in the following circuit:

$$\frac{V_o + V_i + V_o - 10^5 V_i}{50k} = I_o$$

Substituting for $V_i = -\frac{1}{2} V_o$,
$$V_o \left[ \frac{1}{50k} + \frac{1}{250k} \left( \frac{2 \times 10^5}{1 \times 10^3} \right) \right] = I_o$$

$$V_o \left[ \frac{(0.2)10^5}{2 \times 10^5} + \frac{(0.1)10^5}{2 \times 10^5} \right] = I_o$$

Negligible in comparison with $(0.1)10^5$.

$$\therefore \; R_{out} = \frac{V_o}{I_o} = \frac{1}{(0.1)10^5} \approx 0.105 \, \Omega$$
Problem 4: This is HW problem 6.10 re-phrased.
Consider the circuit on the right in which
\( v = 250 \sin(1000t) \) V, \( L = 50 \text{ mH} \), and \( i(0) = -5 \text{ A} \).
Find an expression for the inductor current \( i(t) \).
Also, sketch \( v, i, p, \) power consumed, and energy stored \( w \) in the inductance \( L \) with respect to time \( t \). Label the axes appropriately and show at least one cycle of each variable on your sketch.

\[
\begin{align*}
i &= \frac{1000}{50} \int_0^t 250 \sin 1000z \, dz - 5 \\
&= 5000 \left[ \frac{-\cos 1000z}{1000} \right]_0^t - 5 \\
&= 5(1 - \cos 1000t) - 5 \\
i &= -5\cos 1000t \text{ A} \\
p &= vt = (250 \sin 1000t)(-5\cos 1000t) \\
&= -1250 \sin 1000t \cos 1000t \\
p &= -625 \sin 2000t \text{ W} \\
w &= \frac{1}{2}Li^2 \\
&= \frac{1}{2}(50 \times 10^{-3})^2 25 \cos^2 1000t \\
w &= 625 \cos^2 1000t \text{ mJ} \\
w &= [312.5 + 312.5 \cos 2000t] \text{ mJ}.
\end{align*}
\]
Problem 5: Consider the circuits shown. Explain how and when each circuit acts as a differentiator or an integrator. If proper explanation is lacking, no credit will be given.

The circuit is approximately an integration if $RC > 1$.

$$i = C \frac{dv_o}{dt}$$

$$v_0 = R \cdot i + v_o = RC \frac{dv_o}{dt} + v_0$$

If $RC$ is large, then

$$V_o = RC \frac{dv_o}{dt} \Rightarrow \frac{dv_o}{dt} = \frac{1}{RC} V_o$$

$$V_o(t) = V_0(0) + \frac{1}{RC} \int_0^t V_o(t') \, dt'$$

Assume an ideal op-amp.

Node equation at the negative input terminal:

$$\frac{-V_o}{R} - C \frac{dv_o}{dt} = 0$$

\[ \Rightarrow \frac{dv_o}{dt} = -\frac{1}{RC} V_o \]

Hence $V_o(t) = V_0(t) - \frac{1}{RC} \int_0^t V_o(t') \, dt'$

It is an integration circuit.

Assume an ideal op-amp.

Node equation at the negative input terminal:

$$-C \frac{dv_o}{dt} - V_o = 0$$

\[ \Rightarrow V_o = -RC \frac{dv_o}{dt} \]

It is a differentiator circuit.