The linearized equations governing the motion of a BOEING’s commercial aircraft are given by (Messner and
Tilbury, Control Tutorials for MATLAB and Simulink, Addison Wesley, 1998)\(^1\)

\[
\begin{align*}
\frac{d\alpha(t)}{dt} &= -0.313\alpha(t) + 56.7q(t) + 0.232\delta_e(t) \\
\frac{dq(t)}{dt} &= -0.0139\alpha(t) - 0.426q(t) + 0.0203\delta_e(t) \\
\frac{d\theta(t)}{dt} &= 56.7q(t)
\end{align*}
\] (1)

where \(\theta(t)\) represents the pitch angle. The corresponding open-loop transfer function obtained from (1) is given by

\[
\frac{\Theta(s)}{\Delta(s)} = \frac{1.151s + 0.1774}{s^3 + 0.739s^2 + 0.921s} = \frac{1.151(s + 0.1541)}{s(s^2 + 0.739s + 0.921)} = 1.151G(s)
\] (2)

In this project we design an autopilot that controls the pitch angle \(\theta(t)\) of this aircraft. The autopilot is obtained
by forming the closed-loop system with a unity feedback and a controller of the form \(K_{c}(s)\). For simplicity we
assume that \(K = 1.151K^2\) so that the open-loop feedback transfer function is \(K_{c}(s)G(s)\).

(a) Find the steady state error due to a unit ramp input of the original \((K = 1.151K^2)\) closed-loop system
\((G(s)/1 + G(s))\). Plot the closed-loop system ramp response and observe (check) the corresponding steady
state error. Hint: In order to find the ramp response use the MATLAB function \(y=lsim(cnum,cden,t,t)\) with
\(t=0:0.1:30\).

(b) Find the value for the static gain \(K\) such that the steady state error due to the unit ramp is at most 10\%
\((e_{ss}^{ramp} = 0.1)\). For the obtained value of \(K\) plot the corresponding closed-loop system ramp response and notice
the steady state error improvement. Hint: Use the same time range as in part (a).

(c) For the obtained value of \(K\) find the phase and gain stability margins and observe that the phase margin is
pretty pure. Design the phase-lead controller \(G_{c}(s)\) to improve the phase stability margin such that the compensated
system has the phase stability margin close to 50°. Find the step response of the compensated system and compare
it to the step response of the uncompensated system whose static gain \(K\) is found in part (b). Comment on the
transient response improvement of the compensated system. Hint: In order to be able to estimate the value for \(\omega_{\text{max}}\)
use the following frequency range \(w=0:0.1:0.1:100\) with \(\text{bode}(K\times\text{num},\text{den},w)\). MATLAB will produce, for
this particular example, the Bode plot in the frequency range up to 10 rad/s. However, in the formulated design
problem \(\omega_{\text{max}}\) is greater than 10 rad/s.

(d) Design the phase-lag controller to satisfy the stability requirement imposed in (c). Find the step response of
the system compensated (controlled) by the phase-lag controller and compare it to the step response of the system
compensated by the phase-lead controller. Which one has a smaller rise time? Which one do you prefer?

(e) Using the SIMULINK package, build the block diagrams for the system controlled by phase-lead and
phase-lag controllers, plot the step responses in both cases, and confirm the results obtained in Parts (c) and (d).

Hint: MATLAB programs for Examples 9.4 and 9.5 (posted on the textbook home page).

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\(^{1}\) Note that in the second part of this course we will study the so-called linearization procedure that produces a set of linear differential
equations from a set of nonlinear differential equations.