Digital Electronics Solutions for Homework #2

Question #1:

I_{C(SAT)} = 4.8mA  \quad I_{B(EOS)} = 48\mu A

I_{Bf} = (5-0.7)V/5k\Omega = 860 \mu A  \quad I_{Br} = (0-0.7)V/5k\Omega = -147\mu A

\[ T_s = \tau_o \times \ln \left( \frac{I_{Br} - I_{Bf}}{I_{Br} - I_{B(EOS)}} \right) = 10ns \times \ln \left( \frac{(-140 - 860)\mu A}{(-140 - 48)\mu A} \right) = 16.7ns \]

\[ T_R = \frac{[0.8 Q_A - C_{jC(avg)} \times \Delta V_{BC}]}{[I_{Br} - I_{B(recomo)}]} = \frac{[0.384 pC - 0.4 pC]}{[140\mu A - 24\mu A]} = 4.78ns \]

Where: \( Q_A = \tau_{BF} \times I_{B(eos)} = 10ns \times 48\mu A = 480\mu A \)

\( C_{jC(avg)} = 0.1 pF \quad \Delta V_{BC} = 4V \) (10% to 90%)

\( I_{B(recomo)} = \frac{Q_F}{\tau_{BF}} = \frac{Q_A}{(2*\tau_{BF})} = 24\mu A \)

\[ T_F = \frac{[0.8 Q_A + C_{jC(avg)} \times \Delta V_{BC}]}{[I_{Br} - I_{B(recomo)}]} = \frac{[0.384 pC + 0.4 pC]}{[860\mu A - 24\mu A]} = 0.937ns \]

Part B:

![Waveform Diagram]
Question #2:

a. \( I_C = \frac{V_{cc}}{R_c} = \frac{10V}{1k\Omega} = 10mA \)

\[ I_{B(EOS)} = \frac{I_c}{\beta} = \frac{10mA}{100} = 100\mu A \]

b. \( I_{BF} = \frac{(V_{cc} - V_{BE(sat)})}{R_B} = \frac{(10-0.8)V}{22k\Omega} = 420\mu A \)

c. \( I_{BR} = \frac{(V_{CE} - V_{BE(sat)})}{R_B} = \frac{-0.8V}{22k\Omega} = -36\mu A \)

d. \( Q_A = \tau_{BF}*I_{B(EOS)} = 100\mu A * 20ns = 2pC \)

e. \( Q_s = \tau_s*(I_{BF} - I_{B(EOS)}) = 20ns * 320\mu A = 6.4pC \)

f. \( T_s = \tau_o * \ln \left( \frac{I_{BR} - I_{BF}}{I_{BR} - I_{B(EOS)}} \right) = 20ns * \ln \left( \frac{(-36 - 420)\mu A}{(-36 - 100)\mu A} \right) = 24.196ns \)

g. \( T_R = \frac{[-0.8*Q_A - C_{JC} * \Delta V_{BC}]}{[I_{BR} - I_{B(recombo)}]} = \frac{[-0.8*2pC - 0.1pF * 8V]}{[-36\mu A]} = 66.7ns \)

Where \( \Delta V_{BC} = .1010 \) to \( .9010 = 8V \) and \( I_{B(recombo)} \) is ignored

h. \( I_{BR} = 0 \) in this case so

\[ T_s = \tau_o * \ln \left( \frac{I_{BR} - I_{BF}}{I_{BR} - I_{B(EOS)}} \right) = 20ns * \ln \left( \frac{(0 - 420)\mu A}{(0 - 100)\mu A} \right) = 28.70ns \]

Question #3:

When \( V_i = -\%V \), the BJT is \textbf{OFF}. Therefore, no current is flowing. This means \( V_B = -5V, V_C = 5V \) and \( V_E = 0V \). From the equation sheet we know that
\[
\frac{C_j(V_1)}{C_j(V_2)} = \left( \frac{V_d - V_2}{V_d - V_1} \right)^{1/2}
\]
where \( V_1 \) and \( V_2 \) are the voltages across the voltage in question (either base-emitter or base-collector.  Don't make the mistake of assuming either \( V_1 \) or \( V_2 \) is the potential at some terminal with respect to ground!

Since we already know \( C_{JE(0)} \) and \( C_{JC(0)} \), we can set \( V_2 = 0 \)V and the equation simplifies to:

\[
C_j(V_1) = C_j(0) \left( \frac{V_d - V_2}{V_d - V_1} \right)^{1/2}
\]
Now all we have to calculate are the values of \( V_{(CE)} \) and \( V_{1(C)} \). We can easily observe that \( V_{BE} = -5 \)V and that \( V_{BC} = -10 \)V. Since these are the voltages across the B-E and B-C junctions, they are also \( V_{1(E)} \) and \( V_{1(C)} \).

We can now solve for \( C_{JE(V1E)} \) and \( C_{JC(V1C)} \): Remember, for the \( V_{BE} \) junction, we were given \( V_d = 0.9 \)V.

\[
C_{JE(-5V)} = 1.5 \text{pF} \left( \frac{0.9 - 0}{0.9 - -5} \right)^{1/2} = 0.586 \text{pF}
\]

For the \( V_{BC} \) junction, we were given \( V_d = 0.7 \)V so we calculate:

\[
C_{JC(-10V)} = 0.7 \text{pF} \left( \frac{0.7 - 0}{0.7 - -10} \right)^{1/2} = 0.179 \text{pF}
\]

We know that \( Q = CV \) (\( V \) is still the voltage across the capacitor, not the voltage from a terminal with respect to ground!) So:

\[
Q_{BE} = C_{JE(-5V)} \times -5V = .586\text{pF} \times 5V = -2.93\text{pC}
\]

\[
Q_C = C_{JC(-10V)} \times -10V = .179\text{pF} \times 10V = -1.79\text{pC}
\]
Question #4:

The problem is essentially the same as the previous one, except for one major difference: in this case, the transistor is ON and we can assume that the BJT is saturated! \( I_B = (5-0.7)V/10k\Omega = 0.43mA \) and \( I_{B(EOS)} = I_{C(SAT)}/\beta \)

\[ I_{B(EOS)} = (5-0.2)V/1k\Omega/\beta = 0.048mA. \]

\( I_B > I_{B(EOS)} \) so, yes, we are in saturation.

See the equation sheet for original equations.

\( V_{BE(sat)} \) is roughly 0.7V and \( V_{CE(sat)} \) is roughly 0.2V (given). From here, we can perform the same calculations as we did in Question #3:

\[ C_{jE}(0.7V) = 1.5pF \left( \frac{0.9-0}{0.9-0.7} \right)^{1/2} = 3.18pF \]

\( V_{1C} = V_{BE} - V_{CE} = 0.7V - 0.2V = 0.5V \)

\[ C_{jC}(0.5V) = 0.7pF \left( \frac{0.7-0}{0.7-0.5} \right)^{1/2} = 1.3pF \]

Therefore: \( Q_{BE} = 3.18pF * .7V = 2.22pC \) and \( Q_{CE} = 1.3pF * 0.5V = 0.650pC \)

Question #5:

If the BJT is saturated, the current is:

\( I_c = (V_{cc} - V_{CE(sat)})/R_c = (10-0.2)V/1k\Omega = 9.8mA \)

How do we know where in our range of \( \beta \) to look? Let’s think about it by first calculating \( I_{B(EOS)} = I_{C(sat)}/\beta \). For \( \beta = 30 \), \( I_{B(EOS)} = 9.8mA/30 = 0.327mA \). For \( \beta = 100 \), \( I_{B(EOS)} = 9.8mA/100 = 98\mu A \).

We want the MAXIMUM overdrive factor to be 10 which beats \( 10 \times I_{B(EOS)} = I_B \).
Consider the case where $I_{B(EOS)} = 98\mu A \ (\beta = 100)$. $I_{B1} = 9.8\mu A$ which is too small to drive the transistor with $\beta = 30$ into saturation, let alone into overdrive. It stands to reason, therefore, that we should use the $I_{B(EOS)}$ corresponding to $\beta = 30$ as our $I_{B(EOS)}$ value.

Another way to consider this is to examine the $I_B$ necessary to overdrive the BJT with $\beta = 30$. $I_{B1} = I_{B(EOS)\beta=30} \times 10 = 3.27mA$. This current is large enough to saturate any BJT within the $30 < \beta < 100$ range, but more importantly, it is also large enough to overdrive all the BJTs by at least a factor of 10. (Hence, meeting our requirement of $N_{min} = 10$.)

$I_B = 10*I_{B(EOS)} = 3.27mA = (5 -V_{BE(sat)})V/R_B = (5-0.8)V/R_b$

Therefore $R_B = 4.2V/3.27mA = 1.284k\Omega$

Question #6:

$$T_s = \tau_s \times \ln \left[ \frac{I_{Br} - I_{Bf}}{I_{Br} - I_{B(EOS)}} \right]$$

and $\tau_s = \beta \tau_f = 100 \times 300 \text{ ps} = 30\text{ns}$

Note: All equations come from the Equation Sheet.

We can assume that $V_{BE(sat)} = 0.7V$ and $V_{CE(sat)} = 0.2V$.

$V_{iReverse} = -10V$ in Part A and $V_{ir} = 0V$ for Part B.

$V_{iForward} = 10V$ in Part A & Part B.

Now we can calculate:

<table>
<thead>
<tr>
<th>Calculation</th>
<th>Part A.</th>
<th>Part B.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_{Br} = (V_{iRev} - V_{BE(sat)})/R_B$</td>
<td>$(-10-0.7)V/2k\Omega = -5.35mA$</td>
<td>$(0 - 0.7)V/2k\Omega = -0.35mA$</td>
</tr>
<tr>
<td>$I_{Bf} = (V_{iFor} - V_{BE(sat)})/R_B$</td>
<td>$(10-0.7)V/2k\Omega = $</td>
<td>$(10-0.7)V/2k\Omega = $</td>
</tr>
</tbody>
</table>
\[
I_{B(EOS)} = \frac{I_{C(sat)}}{\beta} = \frac{(V_{CC} - V_{CE(Sat)})}{R_C \cdot \beta}
\]
\[
T_s = \tau_s \cdot \ln \left[ \frac{I_{Br} - I_{Bf}}{I_{Br} - I_{B(EOS)}} \right]
\]
\[
\Delta Q_{BE} = C_{je(\text{av})} \cdot \Delta V_{BE}
\]
\[
\Delta Q_{BC} = C_{jc(\text{av})} \cdot \Delta V_{BC}
\]
\[
\Delta V_{BE} = -10 - 0.5V = -10.5V
\]
\[
\Delta V_{B} = V_{rev} - V_{BE(\text{cut-in})}
\]
\[
C_{jE(\text{av})} = 2C_{jE(0)} / (1 + K_1)^{1/2}
\]
\[
C_{jC(\text{av})} = 2C_{jC(0)} / (1 + K_2)^{1/2}
\]
\[
K_1 = V_{Rev1} / V_{d1}
\]
\[
K_2 = V_{Rev2} / V_{d2}
\]
\[
I_{Bav} = \frac{(I_{B init} + I_{B final})}{2}
\]
\[
I_{B init} = \frac{(V_{i \text{For}} - V_{i Rev})}{R_B}
\]
\[
I_{B fin} = \frac{(V_{i \text{For}} - V_{Be \text{cut in}})}{R_B}
\]
\[
t_D = \frac{(\Delta Q_{BE} + \Delta Q_{BC})}{I_{Bav}}
\]

\[
T_R = \frac{-0.8 \cdot Q_A - C_{jc(\text{av})} \cdot \Delta V_{BC}}{I_{Br} - I_{B\text{(recombo)}}}
\]

\[
\Delta V_{BE} = \Delta V_{B\text{(with emitter grounded)}}
\]

\[
\Delta V_B = V_{rev} - V_{BE(\text{cut-in})}
\]

\[
\tau_{BF} \cdot I_{B(EOS)} = 30ns \cdot 0.208mA = 6.24\ pC\ (\text{Parts A & B})
\]

We need to calculate $I_{B\text{(recombo)}}$, but $C_{jc(\text{av})}$ and $I_{Br}$ have already been calculated.
We need to recalculate \( \Delta V_{BC} \) & \( \Delta V_{BE} \) because previously, we assumed that \( V_C \) and \( V_E \) were both constants and only \( V_B \) changed. Here, \( V_B \) and \( V_E \) are held constant while the output voltage (\( V_C \)) changes from 10% to 90% of its final value.

\[
\Delta V_{BC} = \Delta V_C = V_C(90\%) - V_C(10\%) = 4.5V - 0.5V = 4V \text{ (Both Parts)}
\]

Part A:

\[
T_R = \frac{-0.8 \times 6.24 \, pC - 1.95 \, pF \times 4.0V}{-5.35mA - 0.104mA} = 1.95ns
\]

Part B:

\[
T_R = \frac{-0.8 \times 6.24 \, pC - 1.95 \, pF \times 4.0V}{-0.35mA - 0.104mA} = 23.4ns
\]

And now we must calculate...

\[
T_F = \frac{0.8 \times Q_A + C_{jC(avg)} \times \Delta V_{BC}}{I_{Br} - I_{B(recombo)}}
\]

Part A:

\[
T_F = \frac{0.8 \times 6.24 \, pC + 1.41 \, pF \times 4.0V}{4.65mA - 0.104mA} = 2.34ns
\]

Part B:

\[
T_F = \frac{0.8 \times 6.24 \, pC + 1.95 \, pF \times 4.0V}{4.65mA - 0.104mA} = 2.81ns
\]
In summary:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Part A (-10V-&gt;10V-&gt;-10V)</th>
<th>Part B (0V-&gt;10V-&gt;0V)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_S$</td>
<td>17.62ns</td>
<td>65.79ns</td>
</tr>
<tr>
<td>$T_D$</td>
<td>5.40ns</td>
<td>1.12ns</td>
</tr>
<tr>
<td>$T_R$</td>
<td>1.95ns</td>
<td>23.4ns</td>
</tr>
<tr>
<td>$T_F$</td>
<td>2.34ns</td>
<td>2.81ns</td>
</tr>
</tbody>
</table>

We see that while driving the circuit with a large reverse bias reduces $T_S$, $T_R$ and $T_F$. $T_D$ is larger than driving the circuit with a signal which never reverse biases the base emitter junction.