# Voltage, Current, and Power

The courses Principles of Electrical Engineering I and II are concerned with *Circuit Analysis*. A circuit contains several components called circuit elements or branches. Each circuit element has two or more terminals. When circuit elements are interconnected by having their terminals sharing with each other in a certain manner, we get an interconnected *network* of elements. Hence a circuit is also called a network. Three fundamental variables associated with each element in a circuit are voltage, current, and power. The goal of this notes is to get acquainted with these variables.

We present next the fundamental notions regarding charge, electric field, voltage, current and power.

**Charge:** The concept of electric *charge* is the basis for describing all electrical phenomena. *Charge* is a fundamental physical quantity. Like mass, charge is a property of matter; indeed, charge joins mass, length, time, temperature, and luminosity as one of the fundamental units from which all scientific units are derived. The unit of electric charge is coulomb abbreviated as C. The charge is bipolar; that is electrical effects are described in terms of positive and negative charges. All quantities of charge are integral multiples of $1.6022 \times 10^{-19}$ C. Note that an electron is said to have a negative charge of $1.6022 \times 10^{-19}$ C. *Electric effects are attributed to both the separation of charges and the motion of charges*. The separation of charges creates an electric field that leads to the concept of *electro motive force (emf)* or otherwise called *voltage*, and the motion of charges creates an electric fluid that is commonly called *current*.

**Electric Field:** There exists a force between charged bodies which tends to repel like charges or attract unlike charges. Thus a charge or a set of fixed static charges separated from each other exert a resultant force on a given charged particle. Such a force is called an *electric force*. The *electric field* is a convenient concept in calculating the electric force. Around a charge or a set of fixed static charges we visualize a region of influence called an *electric field*. The electric field intensity or strength $\mathcal{E}$, a vector, is defined by the magnitude and direction of the force $\boldsymbol{f}$ on a unit positive charge in the field. In vector notation, the defining equation is

$$\boldsymbol{f} = q\mathcal{E} \tag{1}$$

where the charge $q$ is measured in coulombs, the magnitude of force $\boldsymbol{f}$ in newtons, and the magnitude of electric field intensity $\mathcal{E}$ in newtons per coulomb. We should note here the analogy between *Electric Field* and *Gravitational Field*.

**Voltage:** The energy-transfering capability of a unit electric charge when it moves from one point to another point in an electric field is said to be the potential difference or electromotive force (emf) or *voltage* between the two points. If an external agent moves a charge of one coulomb from one point to another and in so doing delivers an energy of one joule, then the second point is said to be one volt higher than the first. In general, the potential difference or the voltage between the second point and the first point is $V$ volts given by

$$V = \frac{dw}{dq} \tag{2}$$

where $dw$ in joules (unit abbreviated as J) is the amount of work that needs to be done by an external agent in order to move a charge of $dq$ in coulombs from the first to the second point. We note that a voltage of one volt (abbreviated often as V or $v$) indicates that the amount of work that needs to be done by an external agent is one joule in order to move a charge of one coulomb from the first to the second point.

**Current:** The rate of flow of charge with respect to time is known as the electric *current*, which is expressed as

$$i = \frac{dq}{dt}, \tag{3}$$

where $i$ is the current measured in amperes, $q$ is the charge in coulombs, and $t$ is the time in seconds. We note that a current of one ampere (abbreviated often as A) indicates a flow of one coulomb of charge per second. In general, both positive and negative charges flow through a given cross section. The current through the cross section is then the *net rate* of flow of *positive* charges. Consider a specific case of positively charged particles moving to the right and the negatively charged particles to the left. Then, the net effect of both actions is a positive charge moving to the right. The instantaneous current to the right is given by the equation

$$i = \frac{dq}{dt} = \frac{dq^+}{dt} + \frac{dq^-}{dt} \qquad \left.\begin{array}{l} q^+ \ \oplus \oplus \oplus \oplus \ \rightarrow \\ q^- \ \ominus \ominus \ominus \ominus \ \leftarrow \end{array}\right\} \text{implies that the net current is } \rightarrow$$

where $q^+$ and $q^-$ indicate the <u>magnitudes</u> of positive and negative charges moving respectively to the right and left.

**Power and Energy:** Power is the rate at which work is done with respect to time, or rate at which energy is acquired or given away with respect to time. Mathematically, this can be expressed as

$$p = \frac{dw}{dt}, \tag{4}$$

where $p$ is the power in watts (unit abbreviated as W), $w$ is the work done or energy in joules, and $t$ as usual is the time in seconds. Power is associated with moving charges or currents in an electric field. Indeed, by the chain-rule of calculus, we have

$$p = \frac{dw}{dt} = \frac{dw}{dq}\frac{dq}{dt} = vi \tag{5}$$

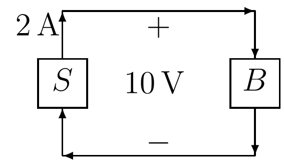
where voltage $= \frac{dw}{dq} = v$ in volts and current $\frac{dq}{dt} = i$ in amperes.

The above equation simply says that the power associated with a circuit element is simply the product of the current $i$ <u>through</u> the element and the voltage $v$ <u>across</u> it. One should be able to tell whether power is delivered by the element or absorbed by the element. By the definition of the voltage, whenever a current goes from the negative side of the voltage to the positive side of the voltage, the <u>agent responsible</u> for such an action delivers power.

*Thus, in a circuit element, if a current $i$ leaves the positive side of the voltage $v$, then the element delivers the power $p = vi$. Conversely, if a curreni enters the positive side of the voltage $v$, then the element absorbs the power $p = vi$.*

Consider the block diagram of a circuit on the right containing two elements S and B. The voltage across each element is 10 volts with the top terminal of each as a positive terminal. On the top side, a current of 2 amperes leaves the element S and enters the element B. On the other hand on the bottom side, a current of 2 amperes leaves the element B and enters the element S. Obviously, a charge of 2 coulombs per second (meaning 2 amperes of current) is pushed by the element S from the bottom side to the top side, *up the potential hill* of 10 volts. This charge of 2 coulombs per second flows through the element B from the top side to the bottom side, *down the potential hill* of 10 volts.

**Anology:** Water pump pushing or pumping fluid around the radiator.




We find that element S has to push 2 coulombs per second up the potential hill of 10 volts. Hence, by the definition of the voltage, element S is supplying (delivering) a power of $10 \times 2 = 20$ watts. In the case of element B, a charge of 2 coulombs per second is flowing down the potential hill of 10 volts. Hence, the element B is consuming (absorbing) a power of $10 \times 2 = 20$ watts. We observe that there is *conservation of power* in a circuit. That is, *whatever power is delivered by certain circuit elements is consumed by the other circuit elements.*

**Reference direction and sign convention:** Before we proceed further, we need to emphasize some fundamental but very elementary notations, reference directions, and sign conventions. This is the basic notation used throughout engineering, mathematics, and other scientific fields. For some students, this discussion is obvious, but others need to understand it critically. We illustrate here the reference directions and sign conventions by two examples. As the first example, consider a checking account in which we can deposit and withdraw money but there is no interest or fees charged. Then the balance of money available to the account holder is given by,

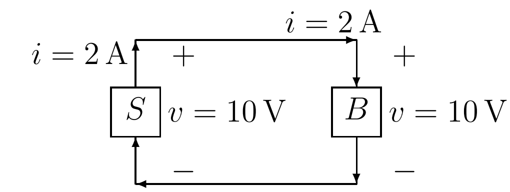
$$x = \text{ Sum of all deposits } - \text{ Sum of all withdrawls.}$$

Note that the variable $x$ is the excess of all the deposits together over all the withdrawls. To exemplify this, suppose $x = \$\,50$. Then, we can say that the bank owes the account holder \$ 50. On the other hand, suppose $x = -\$\,50$. Then, the bank owes the account holder $-\$\,50$ or equivalently we can say that the account holder owes the bank \$ 50. That is, the quantity $x$ can be interpreted in two ways, $x$ is the amount the bank owes to the account holder, or equivalently $-x$ is the amount the account holder owes to the bank. Both the interpretations are mathematically equivalent.

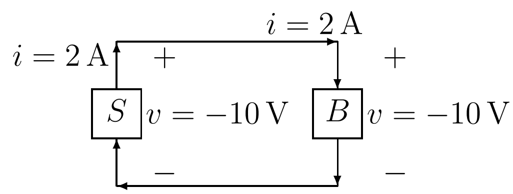
Let us consider next an automobile moving. Suppose we adapt the reference direction as being positive when the automobile moves from south to north. If the automobile has a speed of 30 miles per hour going from south to north, we can say its velocity $v$ is 30 miles per hour. On the other hand, if the automobile has a speed of 30 miles per hour going from north to south, the velocity $v$ is $-30$ miles per hour.

As mentioned above, one must adhere to reference directions and follow accordingly the sign convention. In other words, any scientific analysis uses variables for the concerned quantities with some chosen reference directions. After the analysis, if a certain variable turns out to be negative, then one can interpret this as the corresponding quantity having a magnitude equal to the absolute value of the variable and a direction opposite to the assumed one. The following situations are encountered in connection with power calculations. In all the following four illustrations, **for the indicated reference directions for the voltage as well as current, the power $p = vi$ is consumed by the element $B$, or the power $p = vi$ is generated by the source $S$.** We will concentrate our discussion on element $B$, you could analyze in a similar way what the source $S$ is doing.

In this case, since $v > 0$ and $i > 0$, the power $p = vi$ consumed by the element $B$ turns out to be positive and is given by $p = vi = 20$ W. Indeed, the element $B$ is consuming a power equal to $vi = 20$ W.




In this case, since $v < 0$ and $i > 0$, the power $p = vi$ consumed by the element $B$ turns out to be negative and is given by $p = vi = -20$ W. Actually, the element $B$ is generating a power equal to $|vi| = 20$ W.

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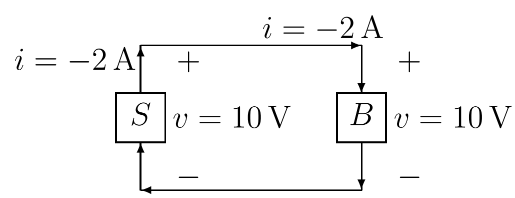
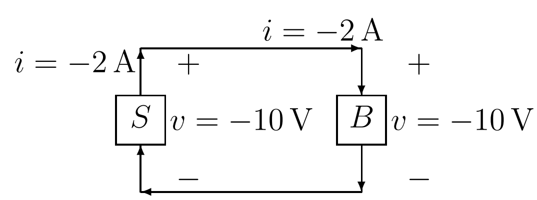



In this case, since $v < 0$ and $i < 0$, the power $p = vi$ consumed by the element $B$ is positive and is given by $p = vi = 20$ W. Thus, indeed, the element $B$ is consuming a power equal to $vi = 20$ W.



## Problems

**1.** A 60W light bulb is rated at 12V.

(a) How much current is flowing through the filament of the bulb? (5 **A which implies** 5 **coulombs of charge per second.**)

(b) What total electric charge does it represent? (18, 000 **Coulombs per hour**)

(c) How many electrons flow through the filament in one hour? ($11.23 \times 10^{22}$ **electrons**) Note that charge on an electron is $e = -1.602 \times 10^{-19}$ Coulombs.

**2.** It is observed that the potential energy lost by an electron in moving from point 1 to point 2 in a circuit is $8 \times 10^{-15}$ Joules. Calculate the potential of point 2 with respect to point 1 and predict the work required to move a charge of $5 \times 10^{-6}$ C from point 1 to point 2 . *Note that in order an electron to lose potential energy in moving from point 1 to point 2, the potential of point 2 must be higher than that of point 1. This is so because electrons are negatively charged particles.* (50 **KV;** 0.25 **Joules**)

**3.** A voltage of 2KV exists across a 1 Cm insulating space between two parallel conducting plates. An electron of mass $m = 9.1 \times 10^{-31}$ Kg is introduced into the space.

(a) Calculate the electric field intensity in the space. (200 **KV/meter**)

(b) Calculate the force and acceleration on the electron. ($3.204 \times 10^{-14}$ **newtons**)

**4.** The charge on one of the plates in problem 3 is known to be $q = -100 \sin(2000t)\mu C$.

(a) Calculate the current in the wire connected to the plate.
($i = -0.2 \cos(2000t)$ **Amperes**)

(b) If the flow of current is due to electrons, in which direction are they moving at $t = 0$? **(Towards the plate)**

**5.** In a TV picture tube, the accelerating voltage is 25 KV.

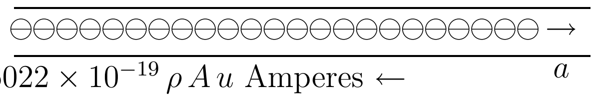
(a) Express in Joules the energy gained by an electron moving through this potential. ($4 \times 10^{-15}$ **Joules**)

(b) If the power delivered to the fluorescent screen by the high velocity electrons is to be 2W, calculate the necessary beam current. ($8 \times 10^{-5}$ **Amperes**)

**6.** A conductor of cross sectional area $A$ meter$^2$ has free electrons whose density is $\rho$ (that is, the number of free electrons per cubic meter is $\rho$). Find the current $i$ in the conductor due to the electron flow from left to right if they are flowing through the conductor with an average velocity of $u$ meters/sec perpendicular to the area of cross section. ($i = 1.6022 \times 10^{-19} \rho A u$ **Amperes**)

Consider the figure. If we could collect all the electrons passing in one second through the point '$a$', we could have collected all the electrons that are in $u$ meters of the conductor. The current flow equals the charge on all the electrons in $u$ meters of the conductor, hence $i = 1.6022 \times 10^{-19} \rho A u$

Amperes flowing from right to left.




$i = 1.6022 \times 10^{-19}\,\rho\,A\,u$ Amperes $\leftarrow$

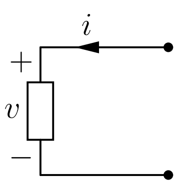
If $A = 4 \times 10^{-12}$ m$^2$, $\rho = 10^{29}$, $u = 156.05$ m/s, find the current $i$. **(**$i = 10$ **Amperes)**

## Summary

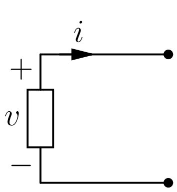
Work is done when a charge moves from one point to another in an electric field.

$q$ = charge (unit is coulomb), $w$ = work done (unit is joule)
$v$ = voltage across two points (electric potential difference between two points)
$= \frac{dw}{dq}$ (unit is volt)
$i$ = current through an element or a branch $= \frac{dq}{dt}$ (unit is ampere)
$p$ = power (rate at which work is done) $= \frac{dw}{dt} = \frac{dw}{dq}\frac{dq}{dt} = vi$ (unit is watt)

Any given circuit consists of several elements or branches each having (at least) two terminals. Each element has a voltage across its two terminals, and current flowing through the terminals. The interaction of voltage and current causes power to be generated or absorbed by the element. Power is generated or absorbed by the element depending upon the reference directions (signs) of the voltage and current as illustrated below.




For the branch or element shown on the left, the current $i$ flows into the positive side of voltage $v$.
The text book by Nilsson & Riedel emphasizes this as a *Passive Sign Convention*. In this case, the power $p = vi$ is always *consumed* by the element. For this case, the charge goes down the potential hill and in so doing loses electric potential; and the released power is absorbed or consumed by the element.
Power **consumed** (absorbed) by the element $= vi$ watts.
We can restate the above as,
power generated (supplied) by the element $= -vi$ watts.
The text book by Nilsson & Riedel always calculates the power consumed by an element, thus in text book notation $p = vi$ watts for the set up shown on the left.




For the branch or element shown, the current $i$ flows out of the positive side of voltage $v$.
For this case, the element pumps the charge up the potential hill, and hence generates power.
Power **generated** (supplied) by the element $= vi$ watts.
We can restate the above as,
power consumed (absorbed) by the element $= -vi$ watts.
The text book by Nilsson & Riedel always calculates the power consumed by an element, thus in text book notation $p = -vi$ watts for the set up shown on the left.