332:221 Principles of Electrical Engineering I – Fall 2006
Hourly Exam 2 – November 6, 2006

Name of the student and ID number:

This is a closed-book closed-notes exam. Do all your work on these sheets. If more space is required, do your work on the back side of the sheets and indicate accordingly so that the grader does not miss it.

Problem #	Page	Maximum Points	Points earned	Description
1	1	13		V_{oc} *by Nodal Analysis*
2	2	14		i_{sh} *by Nodal Analysis*
3	3	13		V_{oc} *by Mesh Analysis*
4	4	15		i_{sh} *by Mesh Analysis*
5	5	20		R_{Th} *by Test Method*
6	6	12		$Op - Amp\ Analysis$
7	7	13		$Op - Amp\ Analysis$

Total maximum points = 100
Total points earned by the student =

Unless you master Node Voltage and Mesh current Methods, you will not shine in Electrical Engineering.

Exam Info: The same circuit is used in all the first five problems. The goal here is to see how well you know Node Voltage Method and Mesh current method, and determining the Thevenin resistance by test voltage method. You need to know super node as well as super mesh concepts. Algebra is simple. In fact you can cross-check answers between different ways of doing the same problem.

(Problem 1) Determination of v_{Th} by node voltage method: Consider the circuit given (not given here but will of course be given in exam). Our aim in this problem is to determine the open circuit voltage v_{oc} at the terminals a and b by utilizing the node voltage method.

(Problem 2) Determination of i_{sh} by node voltage method: Consider the circuit given in Problem 1. Our aim in this problem is to determine the short circuit current i_{sh} through the terminals a and b by utilizing the node voltage method.

(Problem 3) Determination of v_{Th} by mesh current method: Consider the given in Problem 1. Our aim in this problem is to determine the open circuit voltage v_{oc} at the terminals a and b by utilizing the mesh current method.

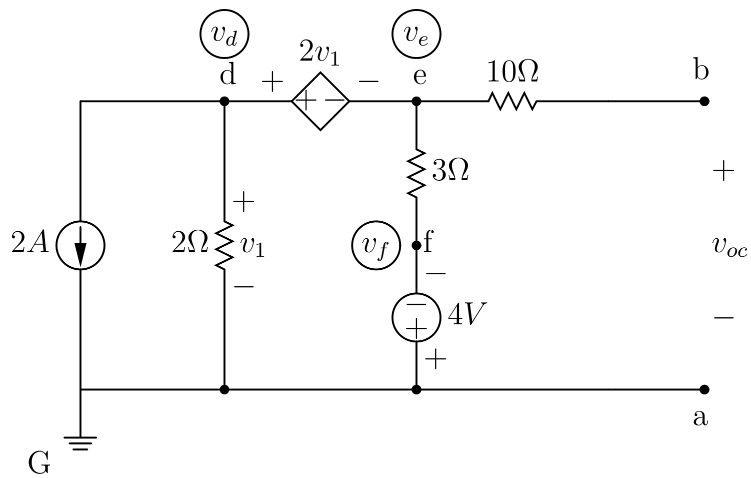
(Problem 4) Determination of i_{sh} by mesh current method: Consider the circuit given in Problem 1. Our aim in this problem is to determine the short circuit current i_{sh} through the terminals a and b by utilizing the mesh current method.

(Problem 5) Determination of R_{Th} by test voltage and test current method: We first construct an independent source-less circuit of the circuit given in Problem 1. Apply a test voltage v_t at the terminals a and b, and determine the test current i_t supplied by it to the circuit. Then we recognize that the Thevenin resistance R_{Th} is the ratio $\frac{v_t}{i_t}$. Utilize any method to determine the ratio $\frac{v_t}{i_t}$.

(Problem 6) This is a problem on Op-Amp circuits.

(Problem 7) This is a problem on Op-Amp circuits.

(Problem 1, 13 points) Determination of v_{Th} by node voltage method: Consider the circuit shown below. Our aim in this problem is to determine the open circuit voltage v_{oc} at the terminals a and b by utilizing the node voltage method.



Consider G as the reference node, and mark the voltages of nodes d, e, and f as v_d, v_e, and v_f as shown. We note that $v_{oc} = v_e$.

Step 1: Write down a relationship that might exist between the voltages v_d and v_1.

We note that $v_d = v_1$.

Step 2: Do we know the value of v_f? If so, what is its value?

We note that $v_f = -4$ V.

Step 3: Write down a relationship that might exist between the node volatages v_d and v_e.

We note that $v_e = v_d - 2v_1 = -v_d$.

Step 4: Write a super node equation involving the nodes d and e.

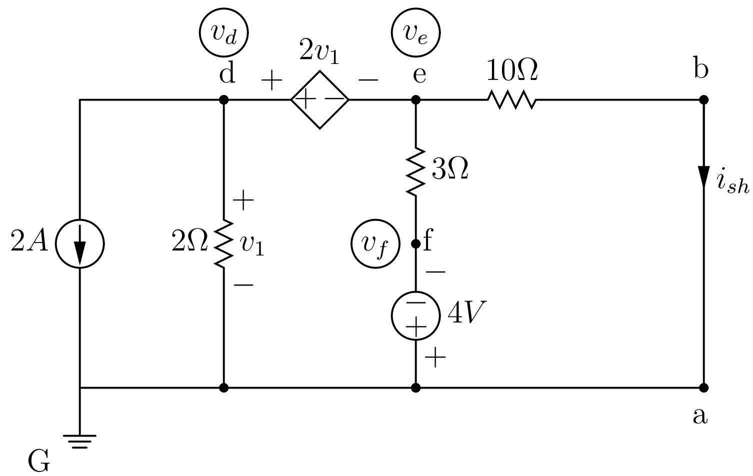
$$2 + \frac{v_d}{2} + \frac{v_e - (-4)}{3} = 0 \quad \Rightarrow \quad 2 + \frac{v_d}{2} + \frac{-v_d + 4}{3} = 0.$$

Step 5: Solve the above equations to determine v_e which equals v_{oc}.
The above implies that

$$v_d = -20\,V \ \text{ and hence } \ v_{oc} = v_e = -v_d = 20\,V.$$

The following was not asked; however it might help to know that $v_{Th} = v_{oc}$.

(Problem 2, 14 points) Determination of i_{sh} by node voltage method: Consider the circuit shown below. Our aim in this problem is to determine the short circuit current i_{sh} through the terminals a and b by utilizing the node voltage method. *Note that some of the steps are same as those in Problem 1, but repeated here for completeness of this problem.*



Consider G as the reference node, and mark the voltages of nodes d, e, and f as v_d, v_e, and v_f as shown.

Step 1: Write down a relationship that might exist between the voltages v_d and v_1.

We note that $v_d = v_1$.

Step 2: Do we know the value of v_f? If so, what is its value?

We note that $v_f = -4$ V.

Step 3: Write down a relationship that might exist between the node volatages v_d and v_e.

We note that $v_e = v_d - 2v_1 = -v_d$.

Step 4: Write a super node equation involving the nodes d and e.

$$2 + \frac{v_d}{2} + \frac{v_e - (-4)}{3} + \frac{v_e}{10} = 0 \quad \Rightarrow \quad 2 + \frac{v_d}{2} + \frac{-v_d + 4}{3} + \frac{-v_d}{10} = 0.$$

Step 5: Solve the above equations to determine v_e.

The above implies that

$$v_d = -50\,V \text{ and hence } v_e = -v_d = 50\,V.$$

Step 6: Once v_e is known, determine i_{sh}.

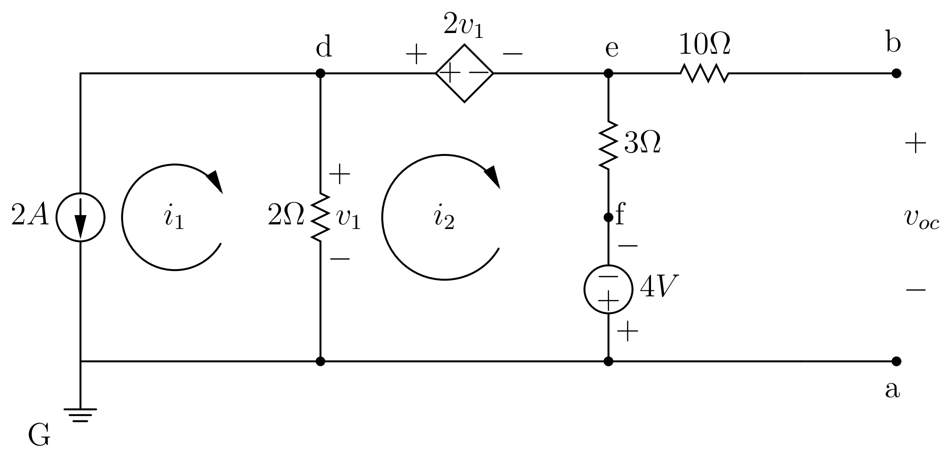
Thus,

$$i_{sh} = \frac{v_e}{10} = \frac{50}{10} = 5\,A.$$

The following was not asked; however it might help to know the answer for Problem 5.

$$R_{Th} = \frac{v_{oc}}{i_{sh}} = \frac{20}{5} = 4\,\Omega.$$

(Problem 3, 13 points) Determination of v_{Th} by mesh current method: Consider the circuit shown below. Our aim in this problem is to determine the open circuit voltage v_{oc} at the terminals a and b by utilizing the mesh current method.



Mark the mesh currents i_1 and i_2 as shown.

Step 1: Do we know the value of i_1? If so, what is its value?

We note that $i_1 = -2$ A.

Step 2: Determine v_1 in terms of i_1 and i_2.

We note that $v_1 = (i_1 - i_2)2$.

Step 3: In view of steps 1 and 2, we need one mesh equation to solve for both the mesh currents. Write an appropriate mesh equation.

We write the mesh equation for i_2 as

$$v_1 - 2v_1 - i_2 3 + 4 = 0 \quad \Rightarrow \quad (i_1 - i_2)2 - 2(i_1 - i_2)2 - i_2 3 + 4 = 0.$$

Step 4: Solve the above equations to determine i_2.

The above implies that

$$i_2 = 8\,A.$$

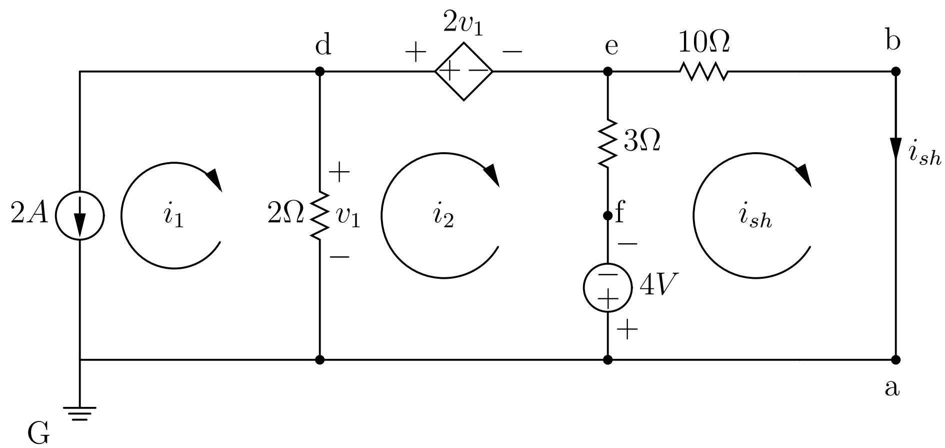
Step 5: Determine $v_{oc} = v_{Th}$ by knowing i_2.

We note that

$$v_{oc} = v_{Th} = -4 + i_2 3 = -4 + 24 = 20V.$$

This agrees with what has been computed in Problem 1.

(Problem 4, 15 points) Determination of i_{sh} by mesh current method: Consider the circuit shown below. Our aim in this problem is to determine the short circuit current i_{sh} through the terminals a and b by utilizing the mesh current method. *Note that some of the steps given below are same as those in Problem 3, but repeated here for completeness of this problem.*



Mark the mesh currents i_1, i_2, and i_{sh} as shown.

Step 1: Do we know the value of i_1? If so, what is its value?

We note that $i_1 = -2$ A.

Step 2: Determine v_1 in terms of i_1 and i_2.

We note that $v_1 = (i_1 - i_2)2$.

Step 3: In view of steps 1 and 2, we need two mesh equations to solve for all the mesh currents. Write appropriate mesh equations.

We write the mesh equation for i_2 as

$$v_1 - 2v_1 - (i_2 - i_{sh})3 + 4 = (i_1 - i_2)2 - 2(i_1 - i_2)2 - (i_2 - i_{sh})3 + 4 = 0 \quad \Rightarrow \quad i_2 = i_{sh}3 + 8\,A.$$

We write the mesh equation for i_{sh} as

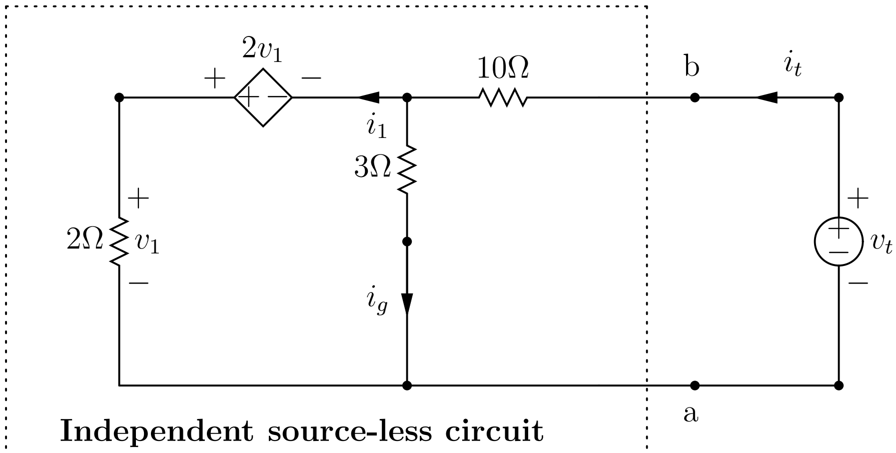
$$-4 - (i_{sh} - i_2)3 - i_{sh}10 = 0 \quad \Rightarrow \quad 13i_{sh} + 3i_2 - 4 = 0.$$

Step 4: Solve the above equations to determine i_{sh}.
Solving the above equations, we get

$$i_{sh} = 5\,A \;\text{ and } i_2 = i_{sh}3 + 8\,A = 23\,A.$$

The value of i_{sh} agrees with what has been computed in Problem 2.

(Problem 5, 20 points) Determination of R_{Th} by test voltage and test current method: We first construct an independent source-less circuit. Apply a test voltage v_t at the terminals a and b, and determine the test current i_t supplied by it to the circuit. Then we recognize that the Thevenin resistance R_{Th} is the ratio $\frac{v_t}{i_t}$. Utilize any method to determine the ratio $\frac{v_t}{i_t}$. The independent source-less circuit along with the test source is as shown below.



We can solve the above problem by utilizing formally the node voltage method or mesh current method. Do by these methods as home-work (not collected). We will solve the problem here by a kind of mixed method (some of the node voltages are mentally simplified).

It is easy to see that

$$i_g = \frac{v_t - i_t 10}{3}.$$

Thus,

$$i_1 = i_t - i_g = i_t - \frac{v_t - i_t 10}{3} \text{ and hence } v_1 = i_1 2 = 2\left(i_t - \frac{v_t - i_t 10}{3}\right).$$

We can now write the KVL equation around the external loop in order to relate v_t and i_t,

$$v_1 - 2v_1 + i_t 10 - v_t = -2\left(i_t - \frac{v_t - i_t 10}{3}\right) + i_t 10 - v_t = 0.$$

We can rewrite the above equation as

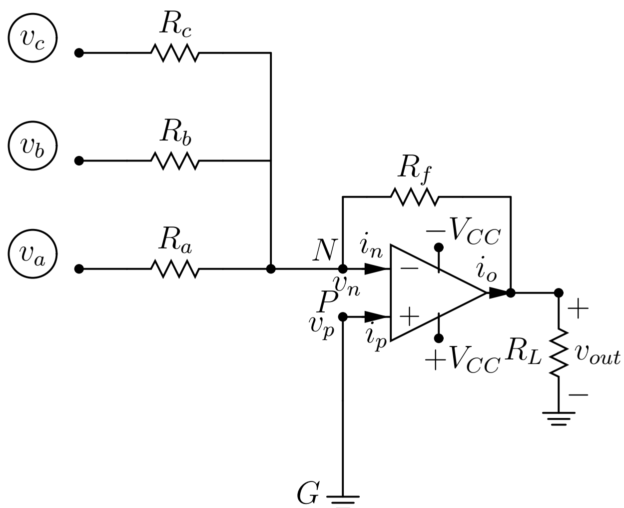
$$\left[-2\left(1 + \frac{10}{3}\right) + 10\right] i_t = v_t \left[1 - \frac{2}{3}\right].$$

By multiplying throughout with 3, the above equation can be simplified as

$$4 i_t = v_t.$$

We can now determine R_{Th} as

$$R_{Th} = \frac{v_t}{i_t} = 4\,\Omega.$$



(Problem 6, 12 points) Consider the **active summing or adder circuit** shown on the left. Design the values for resistances R_a, R_b, R_c, and R_f so that the output voltage v_{out} is the average of v_a, v_b, and v_c except for the sign inversion. That is, we need

$$v_{out} = -\frac{1}{3}(v_a + v_b + v_c).$$

Assume that the Op-Amp is ideal. To get the design equation, write a node equation at the negative input node N of Op-Amp.

Note: In Op-Amp circuits, one should choose resistors large enough not to load the outputs significantly but small enough that stray capacitances do not cause problems. A rule of thumb is to choose resistor values in the range, 500 Ω to 50 KΩ.

Take G as the reference node. Then, to start with, we observe that the node voltages v_n is zero as the Op-Amp is assumed to be ideal. Following the given suggestion, in order to get the design equation, we write a node equation at the negative input node N of Op-Amp as

$$\frac{0 - v_a}{R_a} + \frac{0 - v_b}{R_b} + \frac{0 - v_c}{R_c} + \frac{0 - v_{out}}{R_f} = 0.$$

This can be simplified as

$$v_{out} = -\left[\frac{R_f}{R_a}v_a + \frac{R_f}{R_b}v_b + \frac{R_f}{R_c}v_c\right].$$

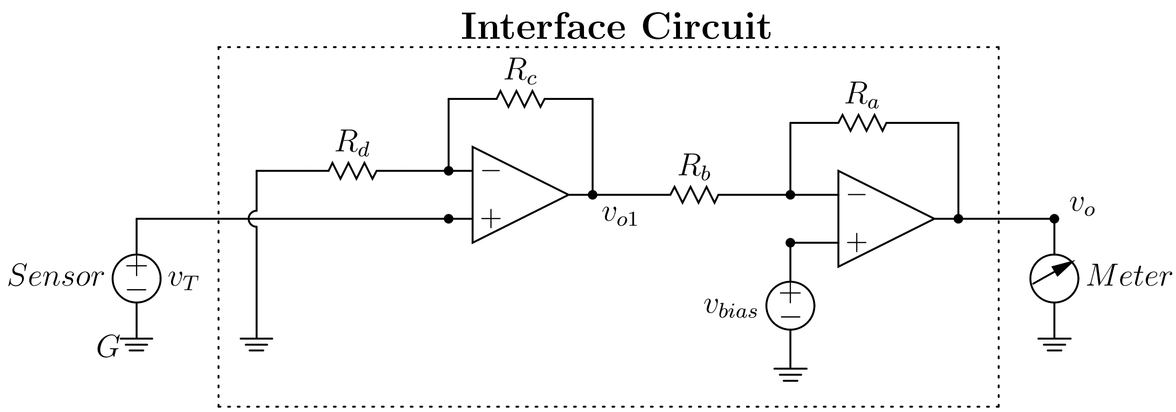
To get the average of all the three signals, we need

$$\frac{R_f}{R_a} = \frac{R_f}{R_b} = \frac{R_f}{R_c} = \frac{1}{3}.$$

That is

$$R_a = R_b = R_c = 3R_f.$$

(Problem 7, 13 points) Consider the following Interface Circuit. Determine the volt-meter reading v_o in terms of the sensor voltage v_T and in terms of all the resistance values and the bias voltage. Assume that the Op-Amps are ideal.



Take G as the reference node. Then, to start with, we observe the following as the Op-Amps are assumed to be ideal:

- The voltage at the negative input terminal of the first Op-Amp is v_T.
- The voltage at the negative input terminal of the second Op-Amp is v_{bias}.

We then write the following node equations one at the negative input terminal of the first Op-Amp and the other at the negative input terminal of the second Op-Amp,

$$\begin{aligned} \frac{v_T}{R_d} + \frac{v_T - v_{o1}}{R_c} &= 0 \\ \frac{v_{bias} - v_{o1}}{R_b} + \frac{v_{bias} - v_o}{R_a} &= 0. \end{aligned}$$

By simplifying the above equations, we get

$$v_o = (1 + \frac{R_a}{R_b})v_{bias} - \frac{R_a}{R_b}(1 + \frac{R_c}{R_d})v_T.$$