332:221 Principles of Electrical Engineering I – Fall 2006

Hourly Exam 1 – October 4, 2006

This is a closed-book closed-notes exam. Do all your work on these sheets. If more space is required, do your work on the back side of the sheets and indicate accordingly so that the grader does not miss it.

Problem #	**Page**	**Maximum Points**	**Points earned**	**Description**
1	1	15		KCL & KVL
2	2	15		KCL & KVL
3	3	25		Tracing & Series-parallel
4	4	20		Solving a simple circuit
5	5	25		Delta - Y transformations

Total maximum points = 100
Total points earned by the student =

Exam Info:

(Problem 1) (15 points) A partially solved circuit is given. You are asked to determine certain voltages and currents by using KCL and KVL.

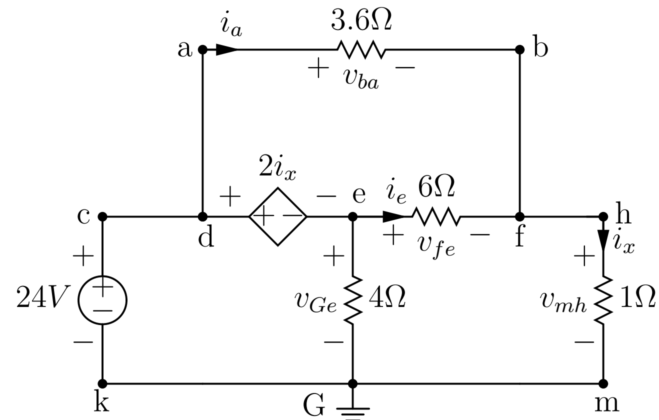
(Problem 2) (15 points) A circuit is given. To solve the circuit, you are systematically guided to write KCL and KVL and then finally to determine the value of an unknown current.

(Problem 3) A circuit and its traced equivalent are given. You need to explain first why the two circuits are equivalent (10 points). Then by series-parallel combinations, you need to simplify the traced circuit and determine the value of an unknown voltage (15 points).

(Problem 4) (20 points) A modified circuit that has been solved in the class is given. You need to solve the circuit.

(Problem 5) (25 points) Using Delta - Y transformations (formulae are provided) you need to first simplify a circuit and then solve it. Current division rule and voltage division rule might help.

Problem 1: (15 points) Consider the so called bridged T circuit that occurs commonly in applications. Some one partially solved the circuit shown on the right and determined v_{Ge} as 12 V. Determine the following by using appropriately, Ohm's law, KVL, and KCL. At each step, state the law you used.



Determine the voltage of the controlled source $2i_x$ and then determine the controlling current i_x. You may write the KVL for the loop kcdeGk.

As suggested, we write the KVL for the loop kcdeGk,

$$24 - 2i_x - 12 = 0 \quad \Rightarrow 2i_x = 12\,V \;\text{ and thus }\; i_x = 6\,A.$$

After knowing i_x, determine v_{mh}.

Using Ohm's law,

$$v_{mh} = (1)i_x = 6\,V.$$

Determine v_{fe}.

Using the KVL for the loop GefhmG,

$$v_{Ge} - v_{fe} - v_{mh} = 0 \quad \Rightarrow v_{fe} = v_{Ge} - v_{mh} = 12 - 6 = 6\,V.$$

Determine i_e.

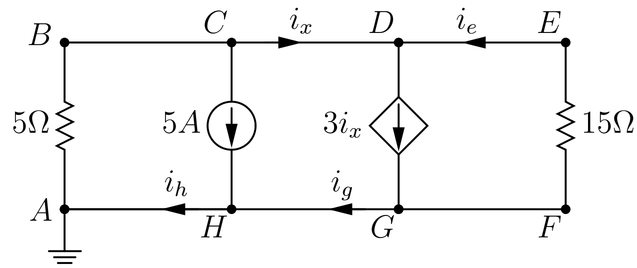
Using Ohm's law,

$$i_e = \frac{v_{fe}}{6} = 1\,A.$$

Determine i_a.
Using the KCL at f,

$$i_a + i_e = i_x = 6 \quad \Rightarrow i_a = i_x - i_e = 6 - 1 = 5\,A.$$

Problem 2: (15 points) Our aim is to determine the current i_x that goes from C to D as shown. Assume that no current flows to the ground. There are a number of ways of doing so. One method follows the steps given below:



Step 1: Determine i_e in terms of i_x by writing the KCL at D or otherwise.

We note that by writing the KCL at D,

$$i_x + i_e = 3i_x \quad \Rightarrow \quad i_e = 2i_x.$$

Step 2: Determine i_g in terms of i_x by writing the KCL at G or otherwise.

We note that

$i_g = i_x$ (This is the cutset equation for the cutset consisting of wires CD and HG).

Or by writing the KCL at G, we get

$$i_g + i_e = 3i_x \quad \Rightarrow \quad i_g = i_x \text{ since } i_e = 2i_x.$$

Step 3: Determine i_h in terms of i_x and 5 A by writing the KCL at H or otherwise.

We note that by writing the KCL at H,

$$i_g + 5 = i_h \quad \Rightarrow \quad i_h = i_x + 5 \text{ since } i_g = i_x.$$

Step 4: Now all the branch currents are written in terms of i_x, write the KVL for the loop ABCDEFGHA.

By writing the KVL for the loop ABCDEFGHA, we get

$$-i_h 5 + i_e 15 = 0 \quad \Rightarrow \quad -(i_x + 5)5 + 2i_x(15) = 0.$$

Step 5: Solve the above equation to determine i_x.

By solving the above equation, we get

$$30i_x - 5i_x = 25 \quad \Rightarrow \quad i_x = 1\,A.$$

Problem 3a: (10 points) Consider the circuit of Figure 1 and explain that it can be re-traced as the circuit given in Figure 2. This can be shown by appropriately naming the nodes in both Figures 1 and 2 (just name the nodes right on the figures) so that Figures 1 and 2 are equivalent.

Problem 3b: (15 points) By appropriately combining the elements, simplify the circuit of Figure 2 to that shown in Figure 3. Evaluate both the resistance values (by using series-parallel combinations) of the circuit of Figure 3. Also, evaluate the value of current source by appropriately combining all the current source values of Figure 2. Then determine the voltage v_1 across the current source. Assume that no current flows to the ground.

We note that 9Ω and 18Ω in parallel is equivalent to 6Ω. Similarly, we note that 15Ω and 7.5Ω in parallel is equivalent to 5Ω. It is then clear that circuit of Figure 3 is a simplified form of circuit of Figure 2. By current division rule, it is easy to see that $i_1 = 2.5$ mA and $i_o = 7.5$ mA. Thus, the voltage v_1 is the voltage across either of the resistances and equals -75 mV. The negative sign is due to the way the direction of v_1 is marked.

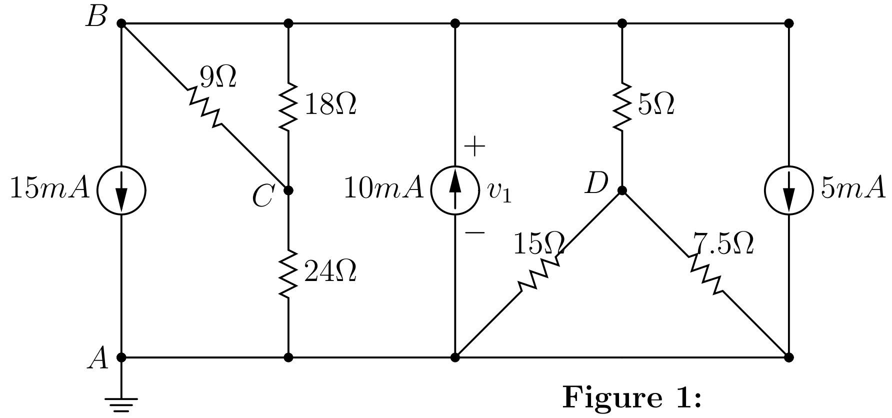


Figure 1:

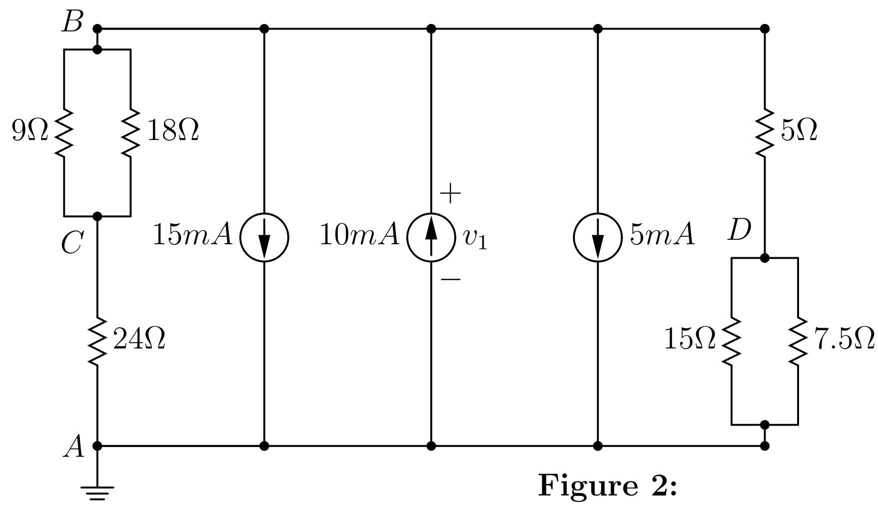


Figure 2:

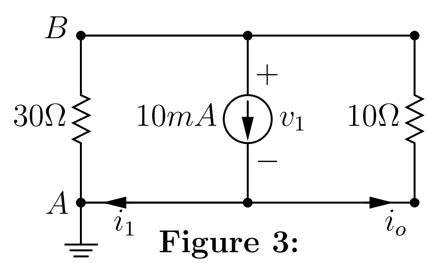
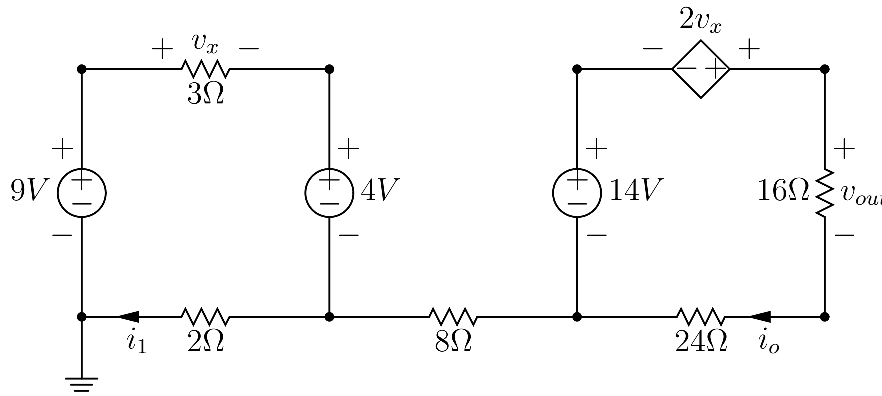


Figure 3:

Problem 4: (20 points) Consider the circuit on the right and determine the output voltage v_{out}. Assume that no current flows to the ground.



By writing the KVL for the left side loop, we get

$$9 - 3i_1 - 4 - 2i_1 = 0 \quad \Rightarrow \quad i_1 = 1\,A.$$

Next, we note that $v_x = 3i_1 = 3\,\text{V}$.
By writing the KVL for the right side loop and knowing $v_x = 3\,\text{V}$, we get

$$14 + 2v_x - 16i_o - 24i_o = 0 \quad \Rightarrow \quad 20 - 40i_o = 0 \quad \Rightarrow \quad i_o = 0.5\,A.$$

This yields us $v_{out} = 16i_o = 8\,\text{V}$.

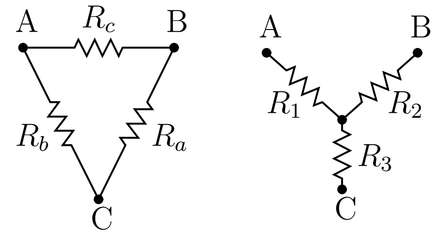
Problem 5: (25 points)

The formulae related to Δ–Y equivalents are given below:

$$R_1 = \frac{R_bR_c}{R_a+R_b+R_c} \qquad R_a = \frac{R_1R_2+R_2R_3+R_3R_1}{R_1}$$

$$R_2 = \frac{R_aR_c}{R_a+R_b+R_c} \qquad R_b = \frac{R_1R_2+R_2R_3+R_3R_1}{R_2}$$

$$R_3 = \frac{R_aR_b}{R_a+R_b+R_c} \qquad R_c = \frac{R_1R_2+R_2R_3+R_3R_1}{R_3}$$



Problem: Our interest in this problem is to determine v_{ba} in the circuit of Figure 1. There are a number of methods of doing so. From what you have learned so far, a viable method is to transform the given circuit of Figure 1 to the circuit of Figure 2 by Δ–Y equivalents, then to the circuit of Figure 3 by series parallel equivalents, and then proceed as necessary. Note that $v_{ba} = v_a - v_b$.

Determine the resistances R_1, R_2, R_3, and R_4.

$$R_1 = \frac{(6)(3)}{6+3+9} = 1\Omega, \quad R_2 = \frac{(6)(9)}{6+3+9} = 3\Omega,$$

$$R_3 = \frac{(9)(3)}{6+3+9} = 1.5\Omega, \quad R_4 = \frac{(10)(6)}{10+6} = 3.75\Omega.$$

Determine the current i from the circuit of Figure 3.

$$i = \frac{40}{4.75+1.5+3.75} = 4A.$$

Knowing the current i, determine the currents i_a and i_b in the circuit of Figure 2 by using the current division rule.

$$i_a = \frac{(4)(6)}{10+6} = 1.5A, \quad i_b = \frac{(4)(10)}{10+6} = 2.5A.$$

Knowing the currents i_a and i_b, determine the voltages v_a and v_b in the circuit of Figure 2.

$$v_a = 9i_a = 13.5V, \quad v_b = 3i_b = 7.5V.$$

Determine $v_{ba} = v_a - v_b$.

$$v_{ba} = v_a - v_b = 13.5 - 7.5 = 6V.$$

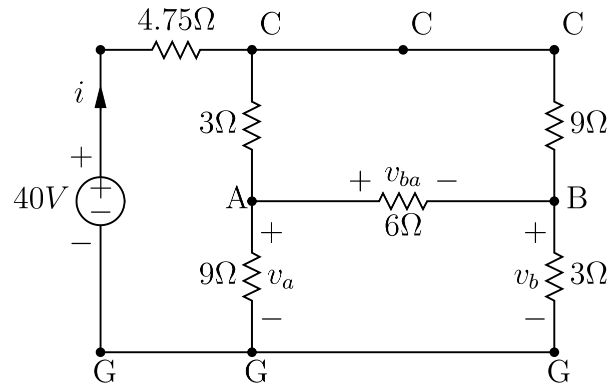


Figure 1

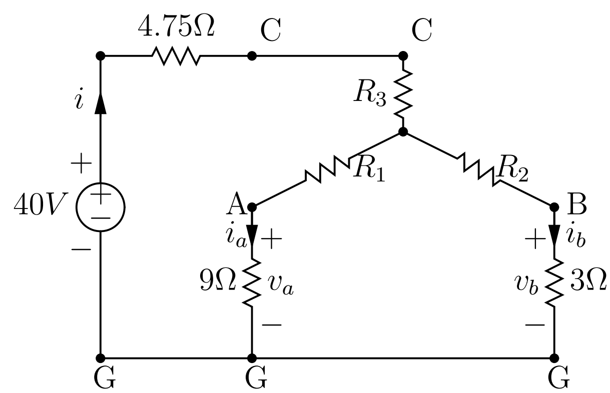


Figure 2

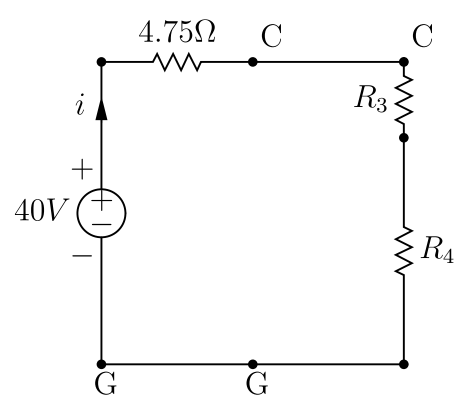
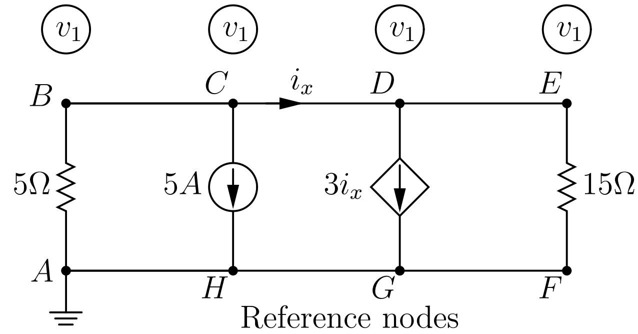


Figure 3

Solution of Problem 2 by node voltage method: Our aim is to determine the current i_x that goes from C to D as shown. Assume that no current flows to the ground.
All the nodes B, C, D, and E are at the same potential. Similarly, all the nodes A, H, G, and F are at the same potential. Select the G as the reference node. Let v_1 be the voltage of top nodes (B, C, D, and E) with respect to the bottom nodes (A, H, G, and F) as marked.



Although all the nodes B, C, D, and E are at the same potential, as we need the current i_x which is in a simple wire, we will split the node into two parts, one part containing B and C and another D and E. Thus, treating B and C as a single node and writing a KCL at that node, we get

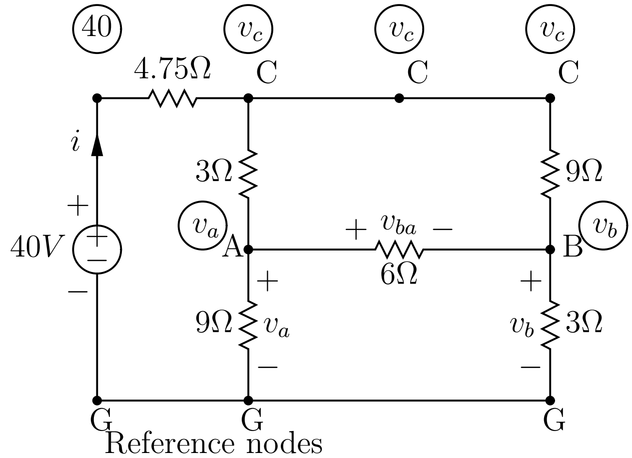
$$\frac{v_1}{5} + 5 + i_x = 0.$$

Similarly, treating D and E as a single node and writing a KCL at that node, we get

$$\frac{v_1}{15} + 3i_x - i_x = 0.$$

By solving the above two equations, we get

$$v_1 = -30\,V \;\text{ and }\; i_x = 1\,A.$$



Solution of Problem 5 by node voltage method: Our interest in this problem is to determine v_{ba} in the circuit given on the right. There are a number of methods of doing so. We will use here node voltage method. Mark the node voltages as shown.

By treating all Cs as a single node, and writing a KCL at that node, we get

$$\frac{v_c - 40}{4.75} + \frac{v_c - v_a}{3} + \frac{v_c - v_b}{9} = 0.$$

By writing a KCL at node A, we get

$$\frac{v_a - v_c}{3} + \frac{v_a - v_b}{6} + \frac{v_a}{9} = 0.$$

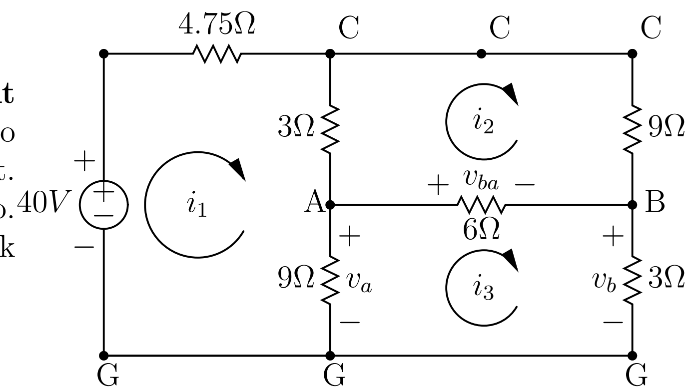
By writing a KCL at node B, we get

$$\frac{v_b - v_c}{9} + \frac{v_b - v_a}{6} + \frac{v_b}{3} = 0.$$

Solving the above three equations, we get

$$v_a = 13.5\,V, \quad v_b = 7.5\,V, \quad v_c = 21\,V.$$

We can now determine v_{ba} as $v_{ba} = v_a - v_b = 13.5 - 7.5 = 6V$.



Solution of Problem 5 by mesh current method: Our interest in this problem is to determine v_{ba} in the circuit given on the right. There are a number of methods of doing so. We will use here mesh current method. Mark the mesh currents as shown.

We can easily write the mesh equations as

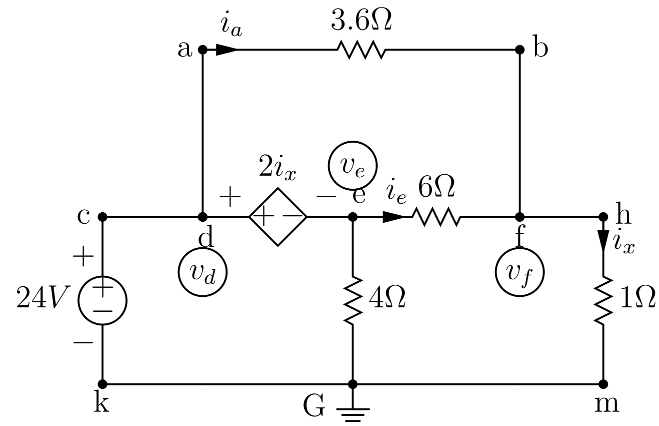
$$\begin{aligned}
40 - 4.75i_1 - 3(i_1 - i_2) - 9(i_1 - i_3) &= 0, \\
-3(i_2 - i_1) - 9i_2 - 6(i_2 - i_3) &= 0, \\
-9(i_3 - i_1) - 6(i_3 - i_2) - 3i_3 &= 0.
\end{aligned}$$

Solving the above three equations, we get

$$i_1 = 4\,A, \quad i_2 = 1.5\,A, \quad i_3 = 2.5\,A, .$$

We can now determine v_{ba} as

$$v_{ba} = 6(i_3 - i_2) = 6V.$$



Solution of Problem 1 by Node-Voltage method: In the exam, we gave v_{Ge} as 12 V (Here we renamed v_{Ge} as v_e). In what follows, we will assume that we do not know v_e, and solve the entire circuit by Node-voltage method.

Let us take G as the reference node. Then, there are three other nodes. We marked their node voltages with respect to G as shown. We note immediately the following.

The node voltage v_d is given and equals 24 V. Also, we note that $v_e = v_d - 2i_x$. Furthermore, we see that $i_x = \frac{v_f}{1} = v_f$. Thus, we have

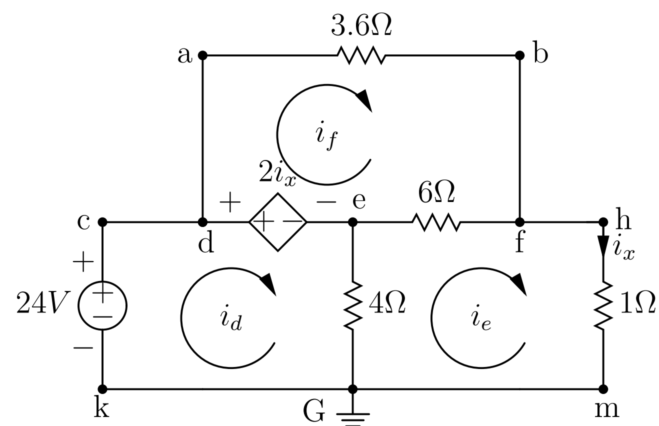
$$v_d = 24\,V \quad \text{and} \quad v_e = v_d - 2i_x = 24 - 2v_f.$$

The above equation reveals that there is only one unknown which we can take as v_f. By writing the node equation at f, we get

$$\frac{v_f}{1} + \frac{v_f - 24}{3.6} + \frac{v_f - (24 - 2v_f)}{6} = 0.$$

We can solve the above equation for v_f which turns out to be 6 V.

Once we know $v_f = 6$ V, we get $v_e = 24 - 2v_f = 12$ V. By knowing thus all the node voltages, we can determine easily the currents and voltages of all branches.



Solution of Problem 1 by Mesh-Current method: In the exam, we gave v_{Ge} as 12 V. In what follows, we will assume that we do not know v_{Ge}, and solve the entire circuit by Mesh-current method.

Three mesh currents are marked as shown. We note immediately that $i_x = i_e$.

We can write three mesh equations as follows:

$$\begin{aligned} 24 - 2i_e - 4(i_d - i_e) &= 0, \\ 2i_e - 3.6i_f - 6(i_f - i_e) &= 0, \\ -i_e - 4(i_e - i_d) - 6(i_e - i_f) &= 0. \end{aligned}$$

We can solve the above equations to get $i_d = 9$ A, $i_e = 6$ A, and $i_f = 5$ A. Once we know these mesh currents, we can determine easily the currents and voltages of all branches.