**NAME IN CAPITAL LETTERS:**

# 332:221 Principles of Electrical Engineering I – Fall 2008
## Hourly Exam 2 – November 3, 2008

This is a closed-book closed-notes exam. Do all your work on these sheets. If more space is required, do your work on the back side of the sheets and indicate accordingly so that the grader does not miss it.

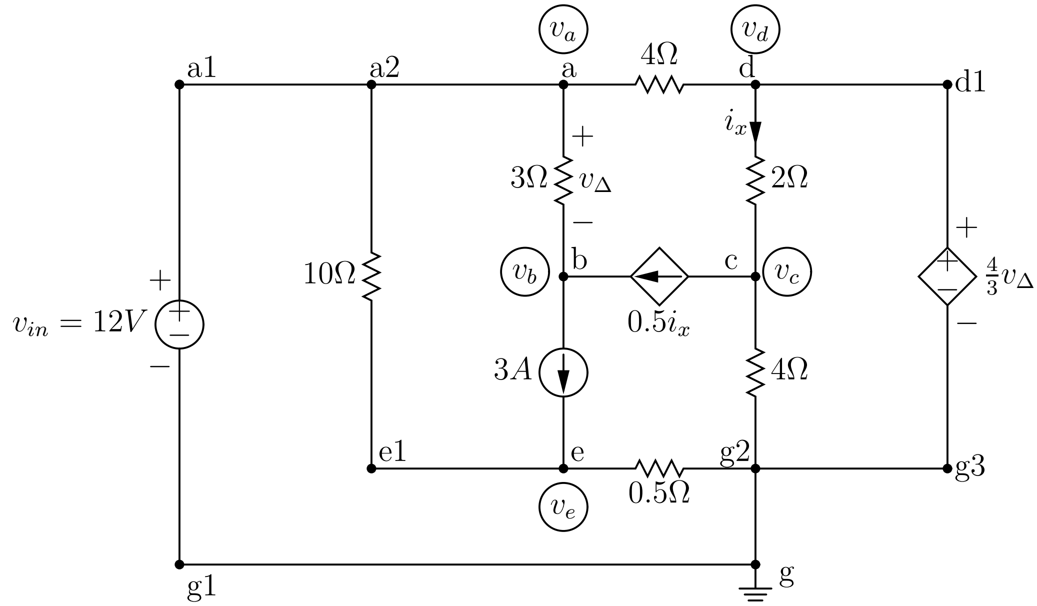
| Problem # | Page | Maximum Points | Points earned | Description |
|---|---|---|---|---|
| 1 | 1 | 20 | | *Node Voltage Method* |
| 2 | 2 | 20 | | *Mesh Current Method* |
| 3 | 3 | 15 | | $V_{oc}$ *by Node Voltage Method* |
| 4 | 4 | 15 | | $i_{sh}$ *by Mesh Current Method* |
| 5 | 5 | 15 | | $R_{Th}$ *by Test Voltage Method* |
| 6 | 6 | 15 | | *Op – Amp Ideal Analysis* |

**Total maximum points = 100**
**Total points earned by the student =**

**Mistakes in sign in setting up the equations will not fetch you any points.**

**Unless you know Node Voltage Method and Mesh current method very well, you will not shine in Electrical Engineering.**

**It is important to note the following:**
**Sources, either voltage sources or current sources, exist in the circuit for some reason. Either they supply power or consume power which is a product of voltage and current. Thus, every source has a voltage across it and current through it.**
**For a voltage source (whether independent or dependent), voltage is prescribed ahead and current is dictated by the circuit depending on its need.**
**For a current source (whether independent or dependent), current is prescribed ahead and voltage is dictated by the circuit depending on its need.**




**Problem 1, 20 points:** We would like to solve the circuit of Figure 1 by **node voltage method**. To do so, we proceed systematically as outlined by the following steps:

**Figure 1**

**Step 1:** We select node g as the reference node, and assign node voltage variables to all the other nodes as shown in circles next to each node in Figure 1. Note that nodes '$a1$' and '$a2$' are the same node '$a$' although they are split up for easiness in drawing the circuit. Similarly, node '$d1$' is same as '$d$'. Also, node '$e1$' is same as '$e$'. Moreover, nodes '$g1$', '$g2$', and '$g3$' are the same node '$g$'.

**Step 2:** Examine both independent and dependent voltage sources and relate them to the basic node voltage variables, $v_a$, $v_b$, $v_c$, $v_d$, and $v_e$.

$$v_a = 12\,V \quad \text{and} \quad v_d = \frac{4}{3} v_\Delta.$$

**Step 3:** Define the controlling variables $i_x$ and $v_\Delta$ in terms of the basic node voltage variables, $v_a$, $v_b$, $v_c$, $v_d$, and $v_e$.

$$i_x = \frac{v_d - v_c}{2} \quad \text{and} \quad v_\Delta = v_a - v_b.$$

**Step 4:** In view of the above steps, how many equations do we have among the variables, $v_a$, $v_b$, $v_c$, $v_d$, and $v_e$? Also, how many more equations do we still need to write in order to enable us to solve all the equations together to determine all the unknown node voltages?

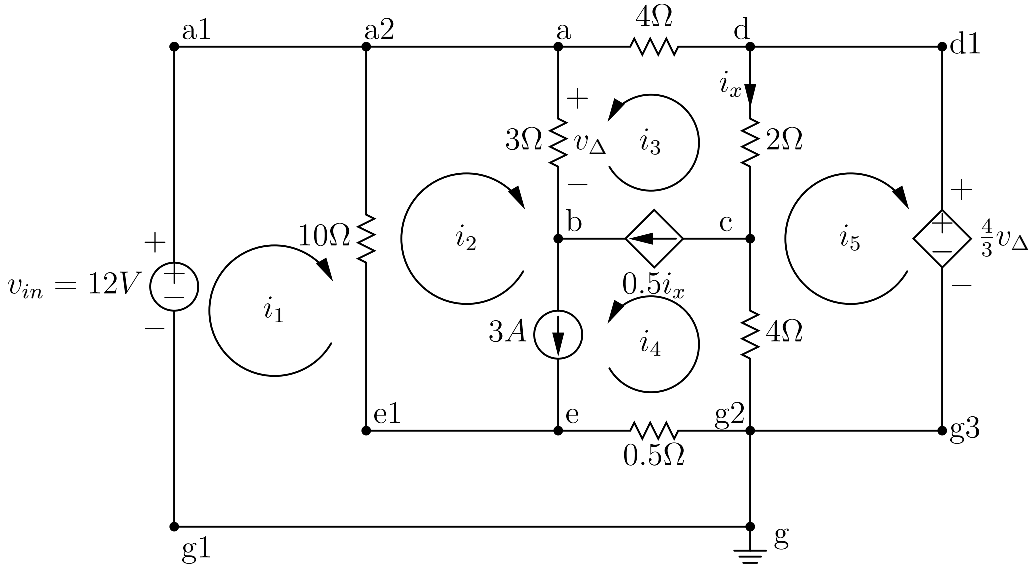
*We have already two equations among the node voltage variables, and we need to write three more equations in order to enable us to solve all the equations together to determine all the unknown node voltages.*

**Step 5:** Write down clearly all the needed number of node equations. **Do not attempt to simplify them or solve them. No supernode equations are necessary.**

$$\begin{aligned} \frac{v_b - v_a}{3} + 3 - 0.5 i_x &= 0 \quad \text{Node equation at node b} \\ 0.5 i_x + \frac{v_c - v_d}{2} + \frac{v_c - 0}{4} &= 0 \quad \text{Node equation at node c} \\ \frac{v_e - v_a}{10} - 3 + \frac{v_e - 0}{0.5} &= 0 \quad \text{Node equation at node e} \end{aligned}$$

Although not asked, the solution of the above equations is given by

$$v_a = 12\,V, \quad v_b = 6\,V, \quad v_c = 4\,V, \quad v_d = 8\,V, \quad \text{and} \quad v_e = 2\,V.$$




**Problem 2, 20 points:** We would like to solve the circuit of Figure 2 by **mesh current method**. To do so, we proceed systematically as outlined by the following steps:

**Figure 2**

**Step 1:** We select the mesh currents $i_1$, $i_2$, $i_3$, $i_4$, and $i_5$ as shown. Note that the mesh current $i_1$ flows around the mesh 'g1 a1 a2 e1 e g2 g g1'.

**Step 2:** Examine both independent and dependent current sources and relate them to the basic mesh currents $i_1$, $i_2$, $i_3$, $i_4$, and $i_5$.

$$i_2 + i_4 = 3\,A \quad \text{and} \quad i_4 - i_3 = 0.5 i_x.$$

**Step 3:** Define the controlling variables $i_x$ and $v_\Delta$ in terms of the basic mesh current variables $i_1$, $i_2$, $i_3$, $i_4$, and $i_5$.

$$i_x = -(i_3 + i_5) \quad \text{and} \quad v_\Delta = (i_2 + i_3)3.$$

**Step 4:** In view of the above steps, how many equations do we have among the variables, $i_1$, $i_2$, $i_3$, $i_4$, and $i_5$? Also, how many more equations do we still need to write in order to enable us to solve all the equations together to determine all the unknown mesh currents?
*We have already two equations among the mesh current variables, and we need to write three more equations in order to enable us to solve all the equations together to determine all the unknown mesh currents.*

**Step 5:** Write down clearly all the needed number of mesh equations. **Do not attempt to simplify them or solve them. One super mesh equation is necessary.**

$$\begin{aligned} 12 - (i_1 - i_2)10 - (i_1 + i_4)(0.5) &= 0 \quad \text{Mesh equation 'g1 a1 a2 e1 e g2 g g1'} \\ -(i_4 + i_5)4 - (i_3 + i_5)2 - \frac{4}{3}v_\Delta &= 0 \quad \text{Mesh equation 'g3 g2 c d d1 g3'} \\ 12 + i_3 4 - \frac{4}{3}v_\Delta &= 0 \quad \text{Super Mesh equation 'g1 a1 a2 a d d1 g3 g g1'} \end{aligned}$$

Although not asked, the solution of the above equations is given by

$$i_1 = 4\,A, \quad i_2 = 3\,A, \quad i_3 = -1\,A, \quad i_4 = 0\,A, \quad \text{and} \quad i_5 = -1\,A.$$

**Problem 3, 15 points:** Consider the circuit shown in Figure 3, and determine the open circuit voltage $v_{\rm OC}$ at the terminals a and b with node b at a higher potential than node a. By using only **Node Voltage Method**, write down all the necessary equations that enable you to solve for $v_{\rm OC}$, and then obtain a numerical value for $v_{\rm OC}$. For easiness in grading your exam, use the notation of node voltages as marked. The reference node is G.

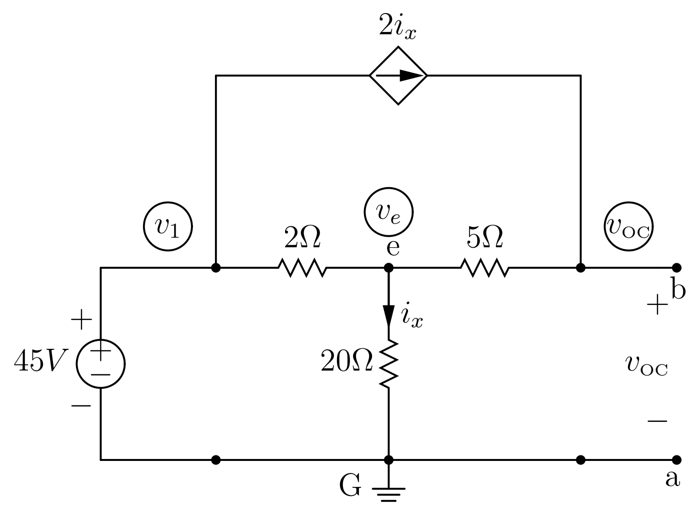



**Figure 3**

**Note:** At first write down all the necessary equations appropriately. To reduce the computational burden in solving them, let me say that $v_e = 50$ V (this is true if you wrote the equations correctly). Use $v_e = 50$ V to solve for $v_{\rm OC}$.

**Solution:** The basic node variables are $v_1$, $v_e$, and $v_{\rm OC}$ as shown in circles at the nodes. We note that

$$v_1 = 45V.$$

Clearly, the controlling variable $i_x$ is given by

$$i_x = \frac{v_e}{20}.$$

We need to write only two node equations, one at node e and the other at node b,

$$\begin{aligned} \frac{v_e}{20} + \frac{v_e - 45}{2} + \frac{v_e - v_{\rm OC}}{5} &= 0 \quad \text{Node equation at node e} \\ \frac{v_{\rm OC} - v_e}{5} - 2i_x &= 0 \quad \text{Node equation at node b} \end{aligned}$$

Substituting for $i_x$ and simplifying the second equation, we get

$$v_{\rm OC} = v_e + 10i_x = v_e + 0.5v_e = 1.5v_e.$$

Substituting for $v_{\rm OC}$ in the Node equation at node e, and simplifying it we get

$$\frac{v_e}{20} + \frac{v_e - 45}{2} + \frac{v_e - 1.5v_e}{5} = 0 \quad \Rightarrow \quad 9v_e = 450 \quad \Rightarrow \quad v_e = 50V.$$

Thus

$$v_{\rm OC} = 1.5v_e = 75V.$$

**Problem 4, 15 points:** Consider the circuit shown in Figure 4, and determine the short circuit current $i_{\text{Sh}}$ from the node b to node a when the terminals a and b are shorted. By using only **Mesh Current Method**, write down all the necessary equations that enable you to solve for $i_{\text{Sh}}$, and then obtain a numerical value for $i_{\text{Sh}}$. For easiness in grading your exam, use the notation of mesh currents as marked.

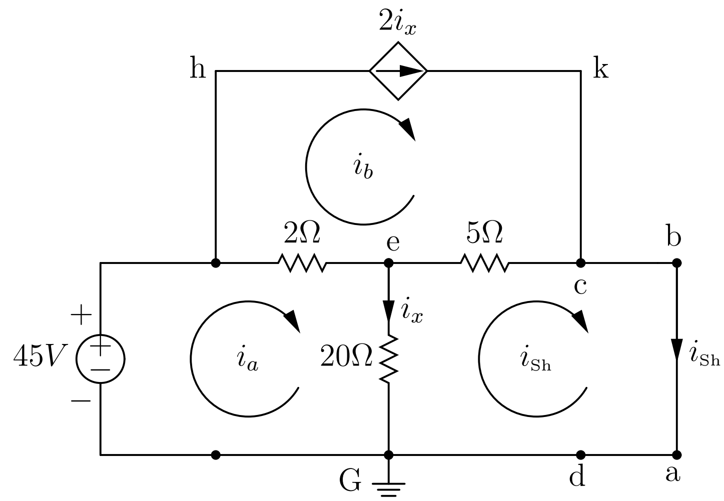



**Figure 4**

**Note:** At first write down all the necessary equations appropriately. To reduce the computational burden in solving them, let me say that $i_a = 10.5$ A (this is true if you wrote the equations correctly). Use $i_a = 10.5$ A to solve for $i_{\text{Sh}}$.

**Solution:** The basic mesh currents are $i_a$, $i_b$, and $i_{\text{Sh}}$ as shown. We note that

$$i_b = 2i_x.$$

Clearly, the controlling variable $i_x$ is given by

$$i_x = i_a - i_{\text{Sh}}.$$

We need to write only two mesh equations as

$$\begin{aligned} 45 - 2(i_a - 2i_x) - 20i_x = 0 \quad &\Rightarrow \quad 18i_a - 16i_{\text{Sh}} = 45 \\ 20i_x - 5(i_{\text{Sh}} - 2i_x) = 0 \quad &\Rightarrow \quad 30i_a - 35i_{\text{Sh}} = 0. \end{aligned}$$

By solving the above equations, we get

$$i_a = 10.5\,A \quad \text{and} \quad i_{\text{Sh}} = 9\,A.$$

**Problem 5, 15 points:** Consider the circuit shown in Figure 3 on page 3. We are interested in determining the Thevenin resistance $R_{Th}$ at the terminals a and b using the test voltage and test current method. For this purpose, a part of the circuit is drawn in Figure 5. **Complete the circuit as needed**, and write down all the necessary equations that enable you to solve for $R_{Th}$. *You do not need to solve the equations, just write them.* Use only **Node Voltage Method**. For easiness in grading your exam, use the notation of node voltages as marked. The reference node is G.

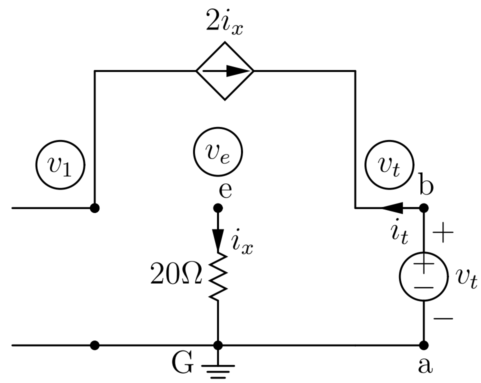



**Figure 5**

**Solution:** Completed circuit is shown in Figure 5a.

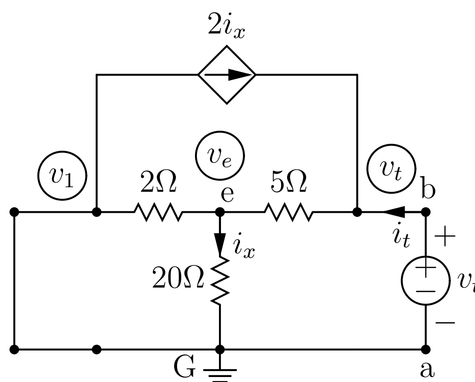



**Figure 5a**

The basic node variables are $v_1$, $v_e$, and $v_t$. Clearly $v_1 = 0$. Also, we assume that $v_t$ is known and compute $i_t$. Moreover, the controlling variable $i_x$ is given by $i_x = \frac{v_e}{20}$. We need to write two equations, one at node e and the other at node b,

$$\begin{aligned} \frac{v_e}{20} + \frac{v_e}{2} + \frac{v_e - v_t}{5} &= 0 \quad \text{Node equation at node e} \\ \frac{v_t - v_e}{5} - 2i_x - i_t &= 0 \quad \text{Node equation at node b} \end{aligned}$$

Assuming $v_t$ is known, we can solve the above two equations for $i_t$. Then, Thevenin resistance $R_{Th}$ is given by $R_{Th} = \frac{v_t}{i_t}$.

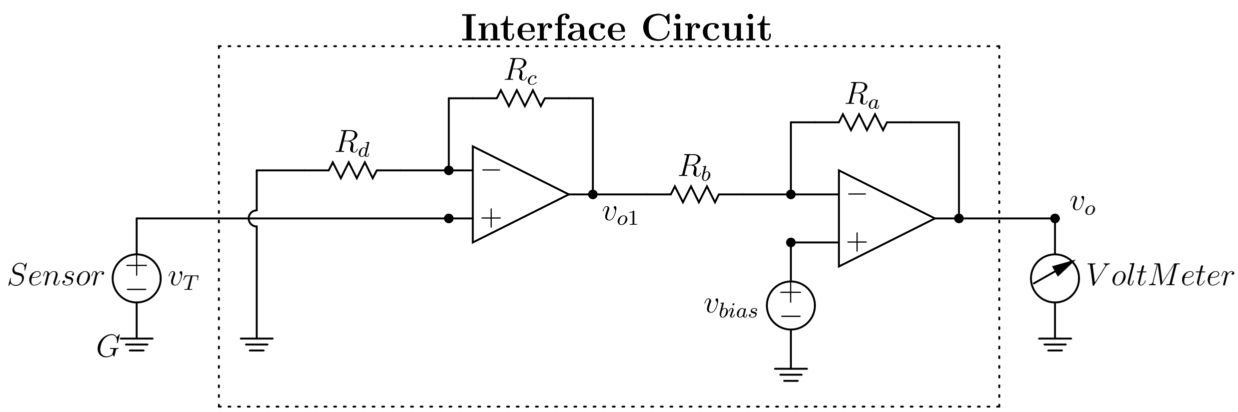
Although, you were not asked to solve the above equations, the above equations can be solved easily. Substituting for $i_x$ and rearranging the above equations, we get

$$\begin{aligned} \frac{v_e}{20} + \frac{v_e}{2} + \frac{v_e - v_t}{5} &= 0 \quad \Rightarrow \quad v_e = \frac{4}{15} v_t \\ \frac{v_t - v_e}{5} - 2i_x - i_t &= \quad \Rightarrow \quad \frac{v_t - v_e}{5} - \frac{v_e}{10} = i_t \end{aligned}$$

Substituting for $v_e$ from the first equation in the second equation and rearranging, we get

$$R_{Th} = \frac{v_t}{i_t} = \frac{25}{3}\Omega.$$

**(Problem 6, 15 points)** Consider the following Interface Circuit. Determine the volt-meter reading $v_o$ in terms of the sensor voltage $v_T$ and in terms of all the resistance values and the bias voltage. Assume that the Op-Amps are ideal.




Take G as the reference node. Then, to start with, we observe the following as the Op-Amps are assumed to be ideal:

- The voltage at the negative input terminal of the first Op-Amp is $v_T$.
- The voltage at the negative input terminal of the second Op-Amp is $v_{bias}$.

We then write the following node equations one at the negative input terminal of the first Op-Amp and the other at the negative input terminal of the second Op-Amp,

$$\begin{aligned} \frac{v_T}{R_d} + \frac{v_T - v_{o1}}{R_c} &= 0 \\ \frac{v_{bias} - v_{o1}}{R_b} + \frac{v_{bias} - v_o}{R_a} &= 0. \end{aligned}$$

By simplifying the above equations, we get

$$v_o = (1 + \frac{R_a}{R_b})v_{bias} - \frac{R_a}{R_b}(1 + \frac{R_c}{R_d})v_T.$$