NAME IN CAPITAL LETTERS:

332:221 Principles of Electrical Engineering I – Fall 2007
Hourly Exam 2 – November 5, 2007

This is a closed-book closed-notes exam. Do all your work on these sheets. If more space is required, do your work on the back side of the sheets and indicate accordingly so that the grader does not miss it.

Problem #	**Page**	**Maximum Points**	**Points earned**	**Description**
1	1	15		V_{oc} *by Nodal Analysis*
2	2	15		i_{sh} *by Mesh Analysis*
3	3	10		R_{Th} *by Node Analysis*
4	4	16		*Nodal Analysis*
5	5	19		*Mesh Analysis*
6*a*	6	10		$Op - Amp\ Ideal\ Analysis$
6*b*	7	15		$Op - Amp\ Non - Ideal\ Analysis$

Total maximum points = 100
Total points earned by the student =

Mistakes in sign in setting up the equations will not fetch you any points.

Unless you know Node Voltage Method and Mesh current method very well, you will not shine in Electrical Engineering.

Problem 1: Consider the circuit shown in Figure 1, and determine the open circuit voltage v_{OC} at the terminals a and b with node b at a higher potential than node a. By using only **Node Voltage Method**, write down all the necessary equations that enable you to solve for v_{OC}, and then obtain a numerical value for v_{OC}. For easiness in grading your exam, use the notation of node voltages as marked. The reference node is G.

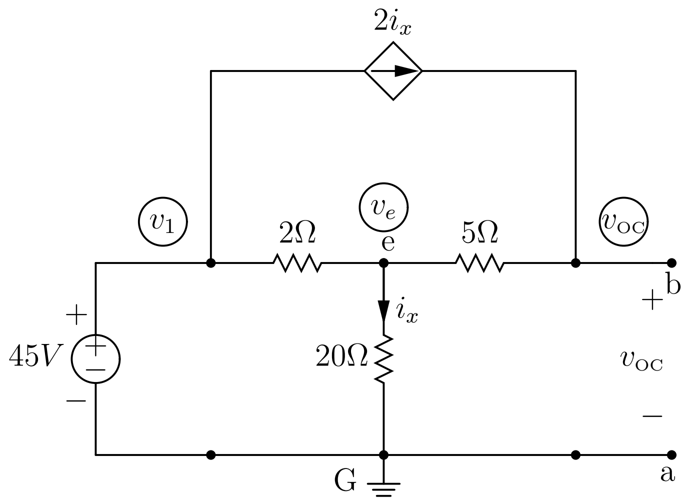

Figure 1

Solution: The basic node variables are v_1, v_e, and v_{OC} as shown in circles at the nodes. We note that

$$v_1 = 45V.$$

Clearly, the controlling variable i_x is given by

$$i_x = \frac{v_e}{20}.$$

We need to write only two node equations, one at node e and the other at node b,

$$\frac{v_e}{20} + \frac{v_e - 45}{2} + \frac{v_e - v_{\text{OC}}}{5} = 0 \quad \text{Node equation at node e}$$

$$\frac{v_{\text{OC}} - v_e}{5} - 2i_x = 0 \quad \text{Node equation at node b}$$

Substituting for i_x and simplifying the second equation, we get

$$v_{\text{OC}} = v_e + 10i_x = v_e + 0.5v_e = 1.5v_e.$$

Substituting for v_{OC} in the Node equation at node e, and simplifying it we get

$$\frac{v_e}{20} + \frac{v_e - 45}{2} + \frac{v_e - 1.5v_e}{5} = 0 \quad \Rightarrow \quad 9v_e = 450 \quad \Rightarrow \quad v_e = 50V.$$

Thus

$$v_{\text{OC}} = 1.5v_e = 75V.$$

Some Wrong equations:

$45 - 2i_x = v_{\text{OC}}$ Incorrect because $2i_x$ is a current source and not a voltage source.

$\frac{45 - v_e}{2} + 2i_x = 0$ Incorrect because the current in the 45V voltage source is ignored.

Problem 2: Consider the circuit shown in Figure 2, and determine the short circuit current i_{Sh} from the node b to node a when the terminals a and b are shorted. By using only **Mesh Current Method**, write down all the necessary equations that enable you to solve for i_{Sh}, and then obtain a numerical value for i_{Sh}. For easiness in grading your exam, use the notation of mesh currents as marked.

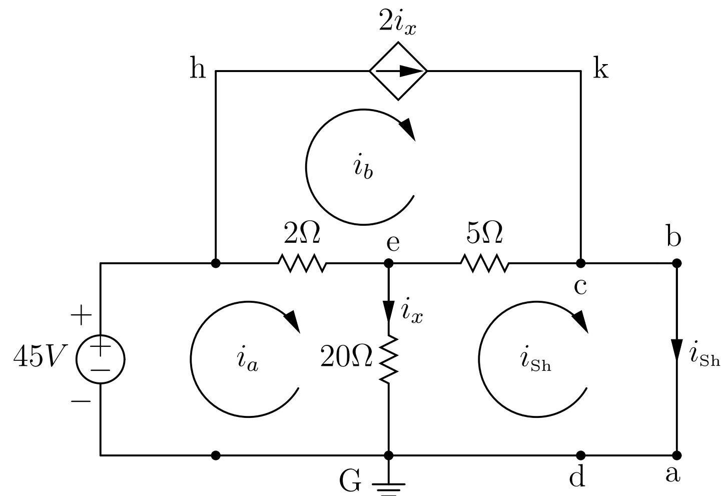

Figure 2

Solution: The basic mesh currents are i_a, i_b, and i_{Sh} as shown. We note that

$$i_b = 2i_x.$$

Clearly, the controlling variable i_x is given by

$$i_x = i_a - i_{\text{Sh}}.$$

We need to write only two mesh equations as

$$\begin{aligned} 45 - 2(i_a - 2i_x) - 20i_x = 0 \quad &\Rightarrow \quad 18i_a - 16i_{\text{Sh}} = 45 \\ 20i_x - 5(i_{\text{Sh}} - 2i_x) = 0 \quad &\Rightarrow \quad 30i_a - 35i_{\text{Sh}} = 0. \end{aligned}$$

By solving the above equations, we get

$$i_a = 10.5\,A \quad \text{and} \quad i_{\text{Sh}} = 9\,A.$$

Some Wrong equations:

$45 \pm 2i_x = 0$ Incorrect because $2i_x$ is a current source and not a voltage source.

$-5(i_b - i_{\text{Sh}}) - 2(i_b - i_a) = 0$ Incorrect because the voltage across the current source is ignored.

$\pm 2i_x - 5(i_b - i_{\text{Sh}}) - 2(i_b - i_a) = 0$ Incorrect because $2i_x$ is a current source and not a voltage source.

Problem 3: Consider the circuit shown in Figure 1 on page 1. We are interested in determining the Thevenin resistance R_{Th} at the terminals a and b using the test voltage and test current method. For this purpose, a part of the circuit is drawn in Figure 3. **Complete the circuit as needed**, and write down all the necessary equations that enable you to solve for R_{Th}. Use only **Node Voltage Method**. You do not need to solve the equations, just set up the equations; do not even simplify them. For easiness in grading your exam, use the notation of node voltages as marked. The reference node is G.

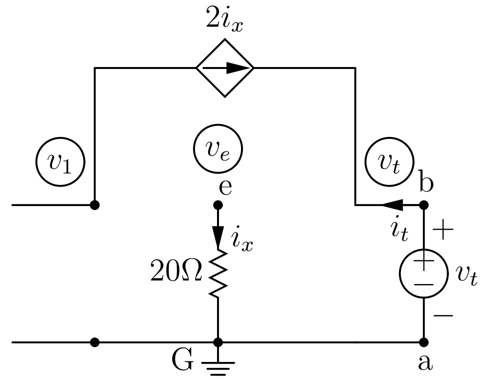

Figure 3

Solution: Completed circuit is shown in Figure 3a.

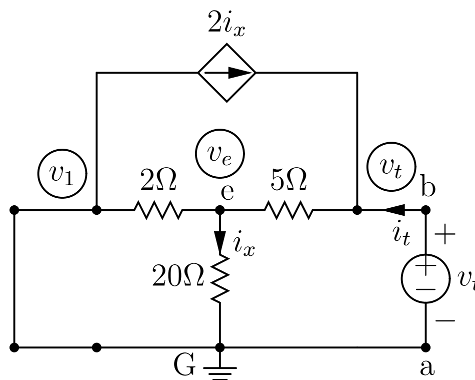

Figure 3a

The basic node variables are v_1, v_e, and v_t. Clearly $v_1 = 0$. Also, we assume that v_t is known and compute i_t. Moreover, the controlling variable i_x is given by $i_x = \frac{v_e}{20}$. We need to write two equations, one at node e and the other at node b,

$$\begin{aligned} \frac{v_e}{20} + \frac{v_e}{2} + \frac{v_e - v_t}{5} &= 0 \quad \text{Node equation at node e} \\ \frac{v_t - v_e}{5} - 2i_x - i_t &= 0 \quad \text{Node equation at node b} \end{aligned}$$

Assuming v_t is known, we can solve the above two equations for i_t. Then, Thevenin resistance R_{Th} is given by $R_{Th} = \frac{v_t}{i_t}$. Although, you were not asked to solve the above equations, the above equations can be solved easily. Substituting for i_x and rearranging the above equations, we get

$$\begin{aligned} \frac{v_e}{20} + \frac{v_e}{2} + \frac{v_e - v_t}{5} &= 0 \quad \Rightarrow \quad v_e = \frac{4}{15} v_t \\ \frac{v_t - v_e}{5} - 2i_x - i_t &= \quad \Rightarrow \quad \frac{v_t - v_e}{5} - \frac{v_e}{10} = i_t \end{aligned}$$

Substituting for v_e from the first equation in the second equation and rearranging, we get

$$R_{Th} = \frac{v_t}{i_t} = \frac{25}{3}\Omega.$$

A number of students made a lot of errors in this problem, it is silly even to repeat them here.

Problem 4: Our interest here is to solve the circuit shown by using only **Node Voltage Method**. Select appropriate basic Node Voltage variables, and write as many equations as there are unknown variables. **You do not need to simplify and solve the written equations**. However, you must write the equations clearly so that some one else can simplify and solve them.

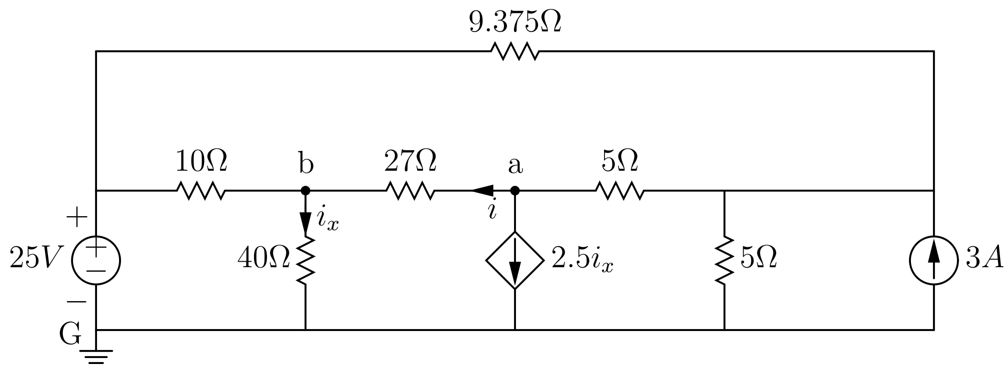

Fig. 1

Solution Select G as the reference node, and set up the node voltages as shown. Write down the needed node equations. Determine first v_a and v_b, and then the current i in 27Ω resistance.

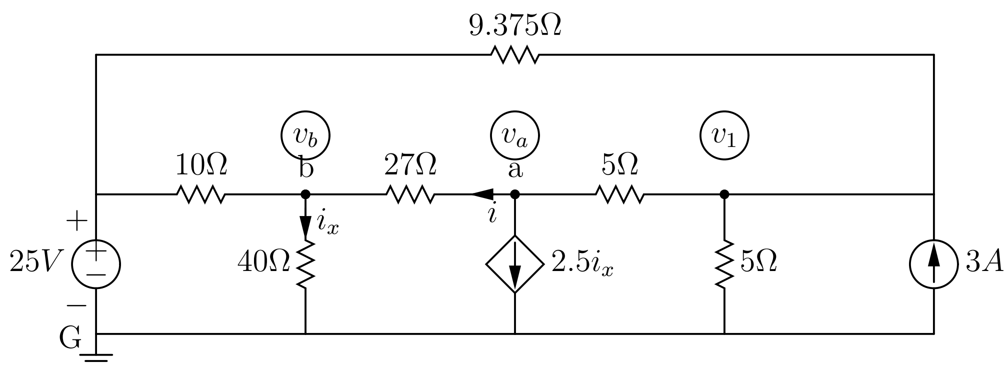

Fig. 2

To start with, we note that $i_x = \frac{v_b}{40}$.
Then the node equations are given by

$$\begin{aligned} \frac{v_1}{5} + \frac{v_1 - 25}{9.375} + \frac{v_1 - v_a}{5} &= 3 \\ 2.5\frac{v_b}{40} + \frac{v_a - v_1}{5} + \frac{v_a - v_b}{27} &= 0 \\ \frac{v_b}{40} + \frac{v_b - 25}{10} + \frac{v_b - v_a}{27} &= 0 \end{aligned}$$

Most of the errors in this problem relate to taking v_1 as zero. Note that there is a voltage across a current source, just like there is a current through a voltage source.

Problem 5: Our interest here is to solve the circuit shown by using only **Mesh Current Method**. Select appropriate basic Mesh Current variables, and write as many equations as there are unknown variables. **You do not need to simplify and solve the written equations**. However, you must write the equations clearly so that some one else can simplify and solve them.

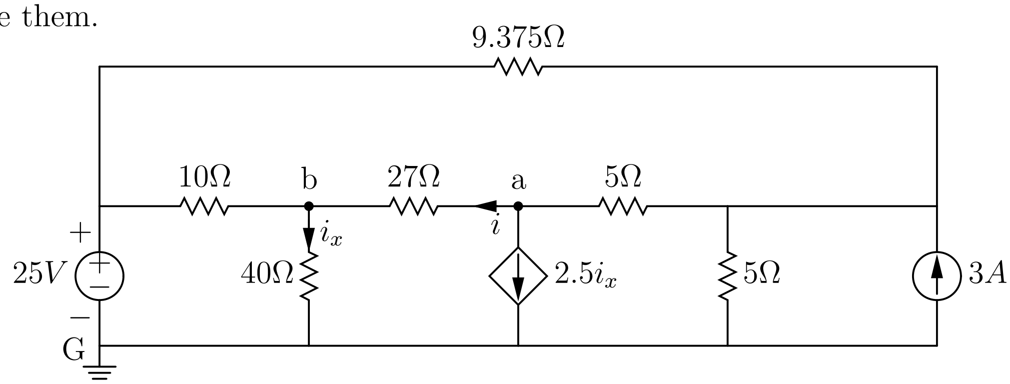

Fig. 1

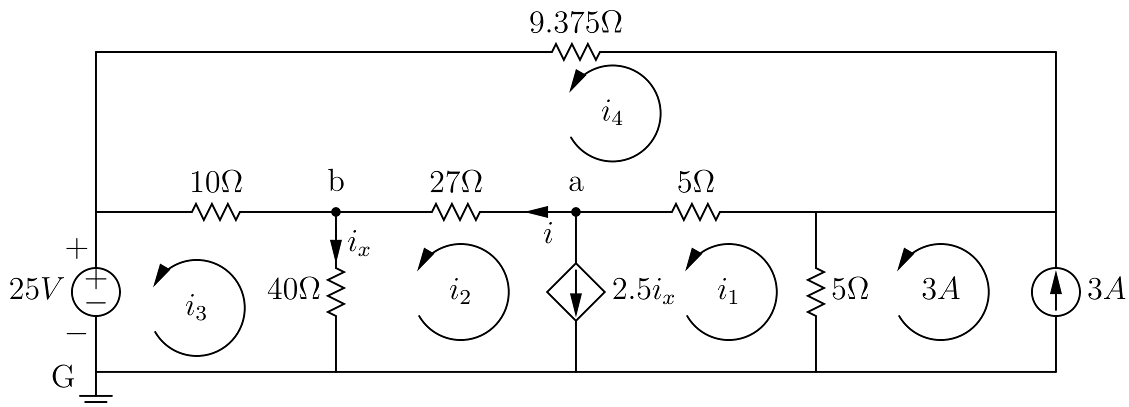

Fig. 3

Solution To start with, we note that $i_x = i_2 - i_3$.
Also, $i_1 - i_2 = 2.5i_x = 2.5(i_2 - i_3)$
(The above equations relate i_1, i_2, and i_3).
Then the remaining three mesh equations are given by

$$\begin{aligned}(i_1 - 3)5 + (i_1 - i_4)5 + (i_2 - i_4)27 + (i_2 - i_3)40 &= 0 \text{ (super mesh)}\\ (i_3 - i_2)40 + (i_3 - i_4)10 + 25 &= 0\\ i_4 9.375 + (i_4 - i_3)10 + (i_4 - i_2)27 + (i_4 - i_1)5 &= 0\end{aligned}$$

Some Wrong equations: A number of students made a lot of errors in this problem, most of the errors relate to either ignoring the voltage across the current sources or using the current as if it is a voltage. Note also that there exists five meshes, and hence five mesh currents must be chosen. Any thing less than five mesh currents means that some mesh currents are inadvertently assumed to be zero.

$(i_1 - 3)5 = 0$ Incorrect because the voltage across the 3 A current source is ignored.

$-40(i_3 - i_2) - 27(i_4 - i_2) = 0$ Incorrect because the voltage across $2.5i_x$ is ignored.

$-5(i_1 - 3) - 5(i_1 - i_4) = 0$ Incorrect because the voltage across $2.5i_x$ is ignored.

$\pm 2.5i_x - 40(i_3 - i_2) - 27(i_4 - i_2) = 0$ Incorrect because the $2.5i_x$ is a current source and not a voltage source.

$\pm 2.5i_x - 5(i_1 - 3) - 5(i_1 - i_4) = 0$ Incorrect because the $2.5i_x$ is a current source and not a voltage source.

Problem 6, 10 points: We considered in the class a voltage divider circuit often. A passive circuit is not appropriate when the load on it changes. An Op-Amp circuit can rectify the problem. In the circuit shown assume that $R_f = R_s$ and $R_1 = 3R_2$ and determine v_{out} in terms of v_g. Assume that the Op-Amp is ideal.

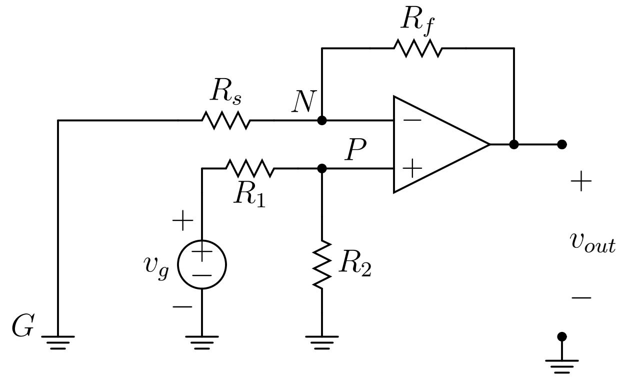

Figure 6

Since Op-Amp is ideal, we know that $v-p = v_n$ (Virtual short at the input end of Op-Amp). Also, $i_p = i_n = 0$ (Virtual open at the input end of Op-Amp).

Node equation at P gives (note that $i_p = 0$),

$$\frac{v_p - v_g}{R_1} + \frac{v_p}{R_2} = 0 \;\Rightarrow\; 4v_p = v_g \,(\text{since } R_1 = 3R_2).$$

This yields $v_p = 0.25v_g$. In fact, this is given by voltage divider principle. Note that $i_p = 0$.

Node equation at N gives (note that $i_n = 0$),

$$\frac{v_n - v_{out}}{R_f} + \frac{v_n}{R_s} = 0 \;\Rightarrow\; v_{out} = 2v_n \,(\text{since } R_f = R_s).$$

Thus, we get

$$v_{out} = 2v_n = 2v_p = 0.5v_g.$$

Problem 7 (15 points), Consider the so called voltage follower Op-Amp circuit shown in Figure 7a on the right. Determine the output voltage v_o and the current i_o supplied by Op-Amp by assuming that Op-Amp is non-ideal with Gain A, input resistance R_{in}, and output resistance R_o. Let $R_L = 100K\Omega$, $R_{in} = 600K\Omega$, $R_o = 1\Omega$, and $A = 10^6$. Neglect any number less than 10^{-4} if it is added or subtracted to a number greater than 1.

Explain all the aspects of your work, otherwise no credit will be given.

To guide you properly, a part of the equivalent circuit is given in Figure 7b where some but not all node voltages with respect to ground are marked.

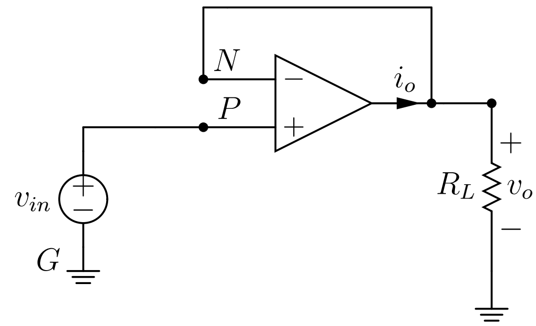

Figure 7a

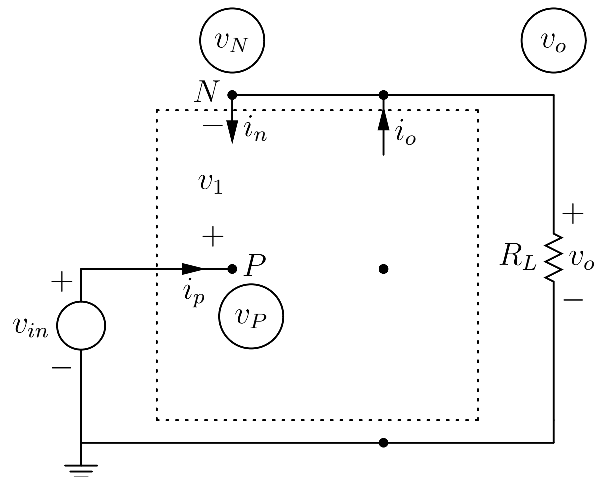

Figure 7b

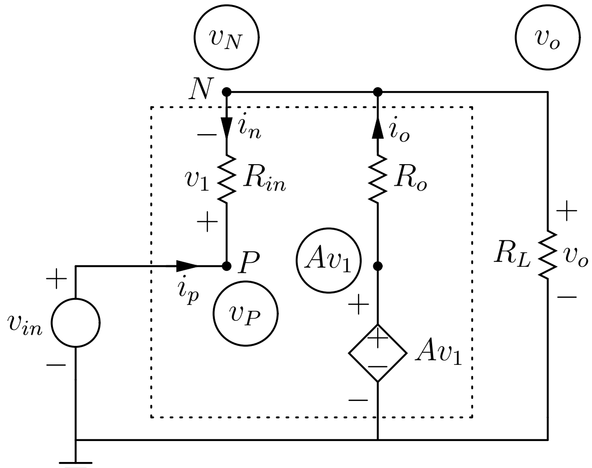

Figure 7c

Solution of Problem 7 by non-ideal analysis: We note easily that $v_P = v_{in}$ and $v_N = v_o$. Also, we note that $v_1 = v_P - v_N = v_{in} - v_o$. By writing the node equation at N, we get

$$\frac{v_o - v_{in}}{R_{in}} + \frac{v_o - A(v_{in} - v_o)}{R_o} + \frac{v_o}{R_L} = 0.$$

The above equation simplifies to

$$\left[\frac{1}{R_{in}} + \frac{1}{R_o} + \frac{1}{R_L} + \frac{A}{R_o}\right] v_o = \left[\frac{1}{R_{in}} + \frac{A}{R_o}\right] v_{in}.$$

For typical values of $R_L = 100K\Omega$, $R_{in} = 600K\Omega$, $R_o = 1\Omega$, and $A = 10^6$, the above equation reduces to
$$\left[\frac{1}{6}(10^{-5}) + 1 + 10^{-5} + 10^6\right] v_o = \left[\frac{1}{6}(10^{-5}) + 10^6\right] v_{in}.$$

In other words, the gain A dominates the equation. As such, it is easy to see that $v_o = v_{in}$. Since $v_P = v_{in}$ and $v_N = v_o$, we note that $v_1 = v_P - v_N = v_{in} - v_o = 0$. This confirms that **the input terminals of Op-Amp are virtually shorted**. The fact that $v_1 = 0$ implies that $i_p = -i_n = 0$. This confirms that **the input terminals of Op-Amp are virtually open**. By writing a node equation at the output end of Op-Amp, we see easily that $i_o = \frac{v_o}{R_L} = \frac{v_{in}}{R_L}$.