332:221 Principles of Electrical Engineering I – Fall 2007
Hourly Exam 1 – October 2, 2008

This is a closed-book closed-notes exam. Do all your work on these sheets. If more space is required, do your work on the back side of the sheets and indicate accordingly so that the grader does not miss it.

Problem #	**Page**	**Maximum Points**	**Points earned**	**Description**
1	1	20		KCL, KVL & Power
2	2	20		Solving a series-parallel circuit
3	3	20		Multiple Choice
4	4	20		KVL of a mesh
5	5	20		KCL at a node

Total maximum points = 100
Total points earned by the student =

Note: Any equation that contains wrong signs for any of its terms is obviously a wrong equation. Not much credit can be given to such an equation. Think of this way. If a deposit in your bank account comes out in the column of your withdrawals, will you complain to the bank or not?

Problem 1: (20 points) Consider the circuit shown. Our aim is to determine i_1, i_2, and i_3 by directly writing the necessary KCL and KVL equations. In order to do the analysis, we divide it into four steps.

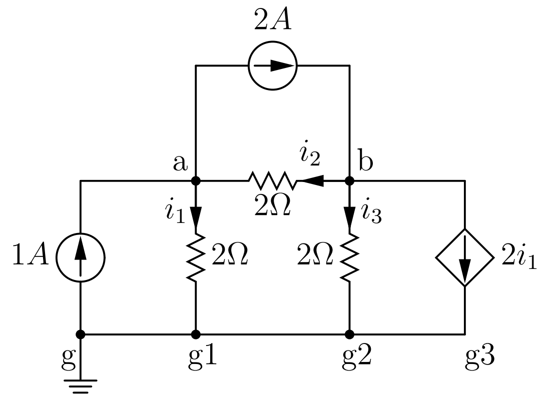


Figure 1

Step 1: Write the KCL at node 'a'.

$$i_1 + 2 - 1 - i_2 = 0.$$

Step 2: Write the KCL at node 'b'.

$$2i_1 - 2 + i_2 + i_3 = 0.$$

Step 3: Write the KVL around the closed-path (loop) '$g1abg2g1$'.

$$2(i_1 + i_2 - i_3) = 0.$$

Step 4, algebra: *Check the above equations carefully once again. If any of the above equations is incorrect, no credit can be given for that equation as well as the following algebra.* Some one who correctly solved the above three equations informed us that $i_1 = 0$ A. In view of this, solve for i_2 and i_3 by making use of any two of the above three equations, and then verify that the solution you obtained satisfies the third equation as well.
Solution: $i_1 = 0$ A, $i_2 = 1$ A, and $i_3 = 1$ A.

Problem 2: (20 points) Consider the circuit shown in Figure 2a. Note that the nodes b, $b1$, and $b2$ are one and the same node. Similarly, the nodes g, $g1$, $g2$, and $g3$ are all one and the same node. For clarity of drawing they were separated out.

We plan to determine certain voltages and currents associated with some branches. To do so, we plan to sequentially simplify the given circuit by series and parallel combinations until we obtain a simple circuit that can be solved easily. Once the reduced simplified circuit is solved we can trace back to obtain the required voltages and currents.

We divide our plan into several steps. **Beware that if you make errors in initial steps, it is hard to grade the following steps and thus you lose points.**

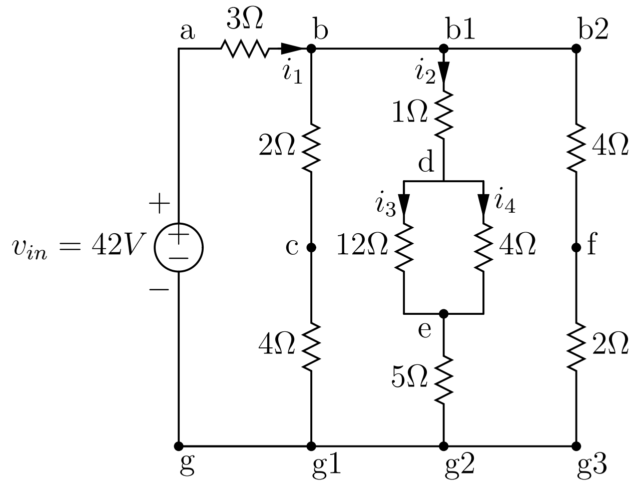


Figure 2a

Step 1: If the circuit of Figure 2a is simplified to that shown in Figure 2b, **determine the values of resistances which are not given in Figure 2b, and mark the values next to them.**

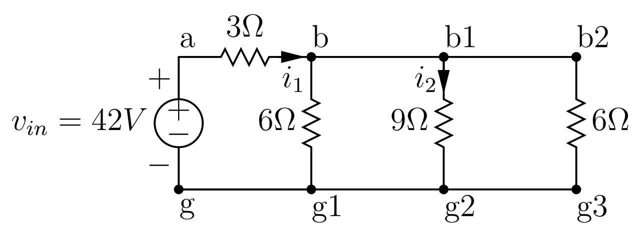


Figure 2b

Step 2: If the circuit of Figure 2b is simplified to that shown in Figure 2c, **determine the value of resistance which is not given in Figure 2c, and mark the value next to it.**

Step 3: In the circuit of Figure 2c, **determine the current i_1 and v_{gb} which is the voltage rise from g to b.**

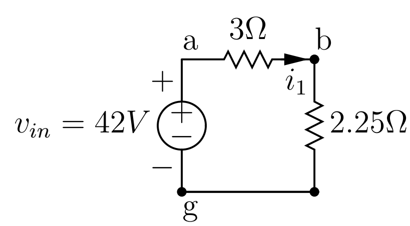


Figure 2c

$$i_1 = 8\,A \qquad v_{gb} = 18\,V$$

Step 4: Move back to the circuit of Figure 2b, and **determine the current i_2.**

$$i_2 = 2\,A$$

Step 5: Move back to the circuit of Figure 2a, and **determine the currents i_3, i_4, and v_{ed} the voltage rise from e to d.**

$$i_3 = 0.5\,A \qquad i_4 = 1.5\,A \qquad v_{ed} = 6\,V$$

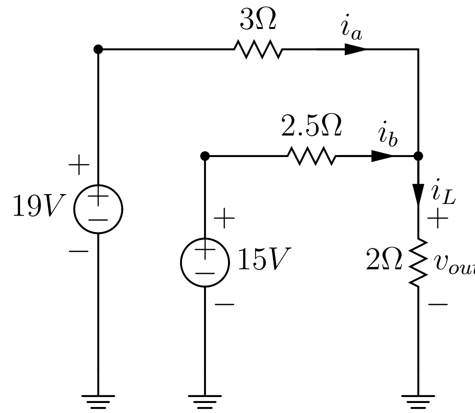
The following are multiple choice problems. Circle one of the answers among the five possibilities given. Explain briefly the reason that led you to the answer you circled.

Note: Think carefully, you do not have to systematically solve the circuit. Consider each answer and check its possibility. A feel for the size of numbers for a given circuit is what is needed to solve multiple choice problems. Such a feel can be obtained by checking one or more KCL or KVL equations in view of the selected answer.

Problem 3a: (10 points) Consider the circuit shown. The output voltage v_{out} is given by

$(a)\ 100\,V\ \ (b)\ 50\,V\ \ (c)\ 10\,V\ \ (d)\ -10\,V$

(e) None of the above

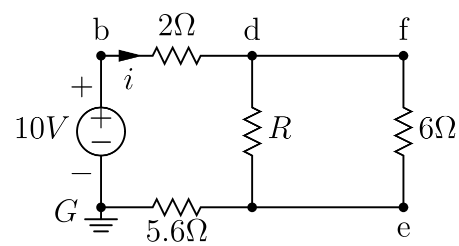


This is a passive circuit. The voltage across any resistance cannot be greater than the sum of source voltages. This is evident by looking at any KVL equation. As such, choices (a) and (b) are not possible. Also, by observing the direction of source voltages, it is easy to infer that v_{out} cannot be negative. This implies that the choice (d) is not possible either. Only the choice (c) is meaningful. Indeed, with the output voltage $v_{out} = 10$ V, we note that $i_L = 5$ A, $i_a = 3$ A, and $i_b = 2$ A. Also, one can note that both KCL and KVL equations are satisfied.

Problem 3b: (10 points) In the circuit shown, the current i delivered by the 10 V source is 1 A. Then, the resistance R must be given by

$(a)\ \infty\,\Omega\ \ (b)\ 100\,\Omega\ \ (c)\ 4\,\Omega\ \ (d)\ 0\,\Omega$

(e) None of the above



$R = \infty$ implies that the parallel equivalent of R and 6Ω is equivalent to 6Ω. This in turn means that the resistance seen by the 10 V source is 13.6Ω. This would lead to a current of $\frac{10}{13.6}$ which is not 1 A.

$R = 100\Omega$ implies that the parallel equivalent of R and 6Ω is equivalent to $\frac{600}{106}\Omega$. This in turn means that the resistance seen by the 10 V source is $2 + 5.6 + \frac{600}{106}\Omega$. This would not lead to a current of 1 A.

$R = 4$ implies that the parallel equivalent of R and 6Ω is equivalent to $\frac{24}{10} = 2.4\Omega$. This in turn means that the resistance seen by the 10 V source is 10Ω. This would lead to a current of 1 A.

$R = 0$ implies that the parallel equivalent of R and 6Ω is equivalent to 0Ω. This in turn means that the resistance seen by the 10 V source is 7.6Ω. This would lead to a current of $\frac{10}{7.6}$ which is not 1 A.

Note: Although the above solutions systematically explain the reasons why one answer could be correct and not the other, all it takes to figure it out is a little bit reasoning of one kind or other.

Problem 4: (20 points) Consider the circuit shown in Figure 4 where each resistance has a value of 0.5 **ohm**. The circuit is solved by what is known as **mesh current method**.

Note that the mesh current is the circulating current around a loop formed by a window whose frames are branches of the circuit. A branch of the circuit which is common to two windows carries two mesh currents, each mesh current corresponding to one window. In this case, the net current in the branch is the algebraic sum of the two mesh currents.

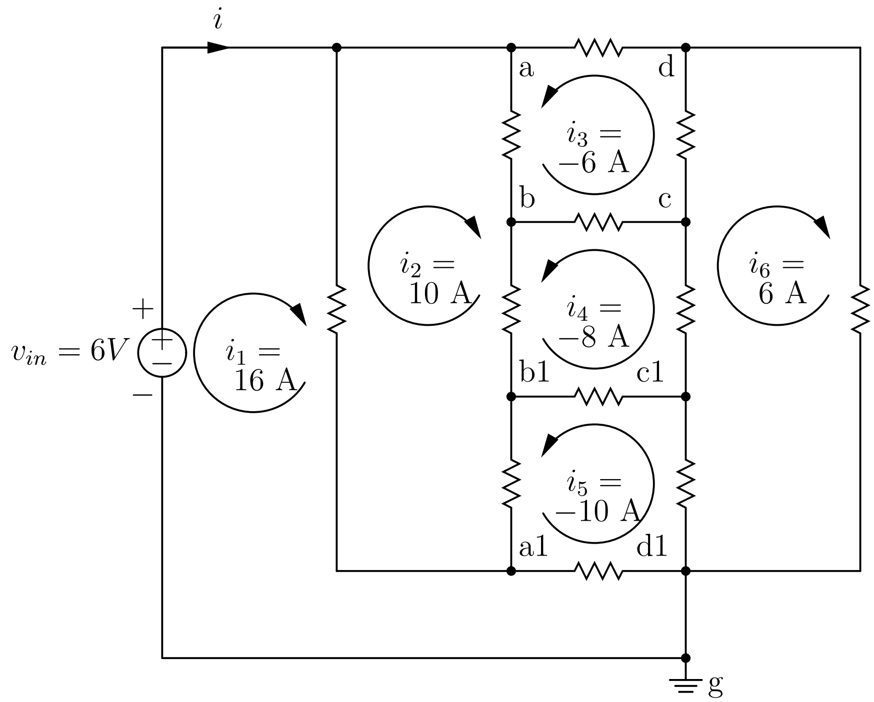


Figure 4

The mesh current method of analyzing the circuit shown yields the solution for the mesh currents as

$$i_1 = 16\,A,\ \ i_2 = 10\,A,\ \ i_3 = -6\,A,\ \ i_4 = -8\,A,\ \ i_5 = -10\,A,\ \ i_6 = 6\,A.$$

Determine the following while indicating appropriate sign (*Beware of Signs*).

- The current in the branch from node $b1$ to node b. **Answer:** $-(i_2 + i_4) = -2\,A$
- The voltage drop from node $b1$ to node b. **Answer: -1 V**
- The current in the branch from node b to node c. **Answer:** $i_3 - i_4 = 2\,A$
- The voltage drop from node b to node c. **Answer: 1 V**
- The current in the branch from node c to node $c1$. **Answer:** $-(i_4 + i_6) = 2\,A$
- The voltage drop from node c to node $c1$. **Answer: 1 V**
- The current in the branch from node $c1$ to node $b1$. **Answer:** $i_5 - i_4 = -2\,A$
- The voltage drop from node $c1$ to node $b1$. **Answer: -1 V**
- **In view of the above calculations, verify that the Kirchoff's Voltage Law is satisfied around the loop** $b1\,b\,c\,c1\,b1$. Answer: Yes, KVL is satisfied.

Problem 5: (20 points) A dozen resistors each having a resistance of 0.5 **ohm** form the arms of a cube. For easiness and for clarity in drawing, the resulting circuit is drawn on a single plane as shown. A voltage of six volts is applied between the nodes a and d_1 as shown in Figure 5. The resulting circuit is solved by what is known as **node voltage method**. The analysis yields the voltage of each node with respect to a reference node. For this example, node d_1 is used as the reference node. With respect to node d_1, the voltage of each node is marked as shown in a circle next to the node. These node voltages are like measured heights of several points on a hill, each height being measured with respect to a reference point.

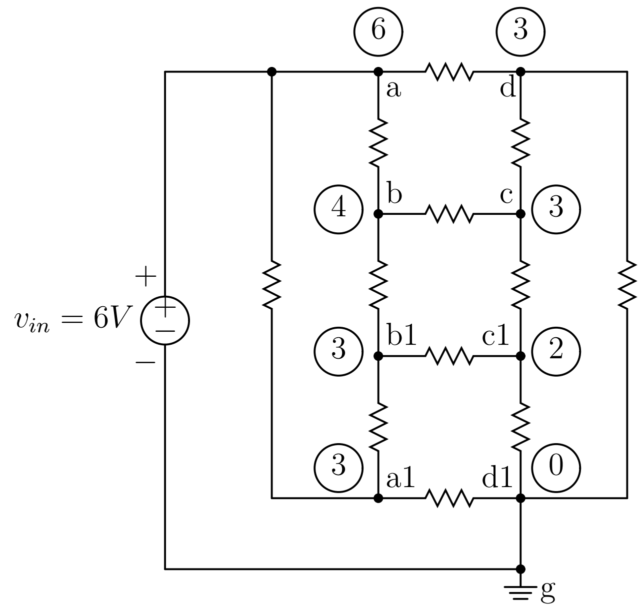


Figure 5

Determine the following while indicating appropriate sign (*Beware of Signs*).

- The voltage rise from node b_1 to node b. **Answer: 1 V**
- The current from node b_1 to node b. **Answer: -2 A**
- The voltage rise from node a to node b. **Answer: -2 V**
- The current from node a to node b. **Answer: 4A**
- The voltage rise from node c to node b. **Answer: 1 V**
- The current from node c to node b. **Answer: -2 A**
- **In view of the above calculations, verify that the Kirchoff's Current Law is satisfied at node** b. Answer: Yes, KCL is satisfied.