

332:221 Principles of Electrical Engineering I – Fall 2007

Hourly Exam 1 – October 3, 2007

This is a closed-book closed-notes exam. Do all your work on these sheets. If more space is required, do your work on the back side of the sheets and indicate accordingly so that the grader does not miss it.

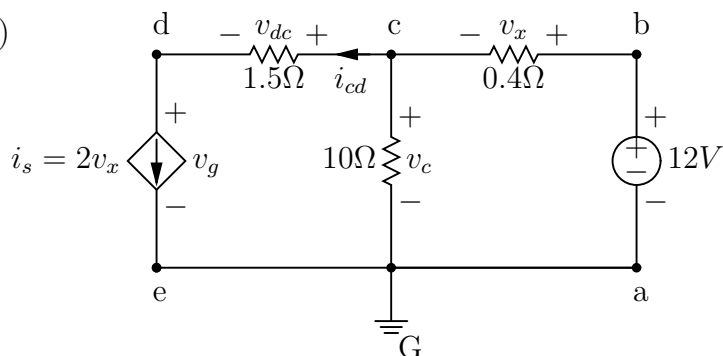
Problem #	Page	Maximum Points	Points earned	Description
1	1	20		KCL, KVL & Power
2	2	15		KCL, KVL & Power
3	3	20		Voltage divider
4	4	20		Solving a simple circuit
5	5	25		Delta - Y transformations

Total maximum points = 100

Total points earned by the student =

Note: Any equation that contains wrong signs for any of its terms is obviously a wrong equation. Not much credit can be given to such an equation. Think of this way. If a deposit in your bank account comes out in the column of your withdrawals, will you complain to the bank or not?

Problem 1: (20 points)



Some one partially and correctly solved the circuit shown above and determined v_c as 10 V. With this knowledge, determine the following:

(a) Determine v_x and then the current $i_s = 2v_x$ of the controlled source. To determine v_x you may make use of the KVL around the loop abcGa.

The KVL around the loop abcGa is given by

$$12 - v_x - v_c = 0 \Rightarrow v_x = 12 - v_c = 12 - 10 = 2 \text{ V.}$$

Thus the the current $i_s = 2v_x = 4 \text{ A}$.

(b) What is the current i_{cd} that flows through the 1.5Ω resistance? To determine this you may make use of the KCL at the node d.

We note that $i_{cd} = i_s = 4 \text{ A}$.

(c) Once the current i_{cd} is known, determine the voltage v_{dc} .

We note that $v_{dc} = i_{cd}1.5 = 6 \text{ V}$.

(d) Determine v_g , the voltage of the controlled source. To determine v_g you may make use of the KVL around the loop GcdeG.

The KVL around the loop GcdeG is given by

$$v_c - v_{dc} - v_g = 0 \Rightarrow v_g = v_c - v_{dc} = 10 - 6 = 4 \text{ V.}$$

(e) Determine the power associated with the controlled source. Is this power consumed or generated by the controlled source?

The power consumed by the controlled source = $v_g i_s = (4)(4) = 16 \text{ W}$.

Problem 2: (15 points) Consider the circuit shown. By writing a KCL at an appropriate node, determine i_s .

By writing a KCL at node f, we get $i_s = 5$ A.

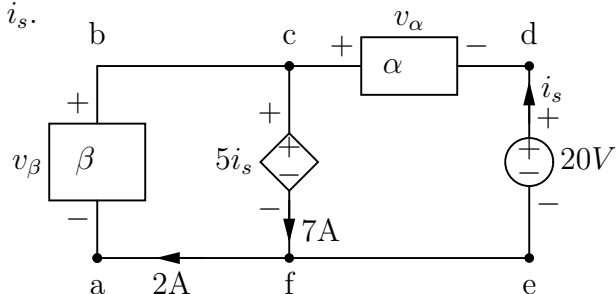


Figure 1

Determine the voltage across the dependent source in the direction shown.

The voltage across the dependent source is $5i_s = 25$ V.

By writing a KVL around an appropriate loop, determine the voltage v_α across the element α in the direction shown.

The voltage v_α across the element α can be obtained by writing the KVL around the closed path cdefc as 5 V.

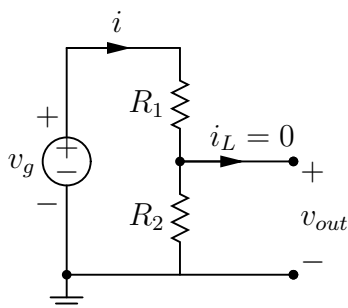
By writing a KVL around an appropriate loop, determine the voltage v_β across the element β in the direction shown.

The voltage v_β across the element β can be obtained by writing the KVL around the closed path abcfa as 25 V.

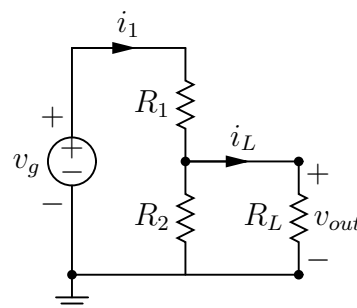
Fill the following table regarding power consumption or generation. A particular element either consumes or generates power. You must write the appropriate value for a particular element in **only one** of the unfilled columns whichever is appropriate.

Element	Power Consumed	Power Generated
Element α		$5 \times 5 = 25$ Watts
Element β		$2 \times 25 = 50$ Watts
Independent Source		$5 \times 20 = 100$ Watts
Dependent Source	$7 \times 25 = 175$ Watts	

Problem 3: (20 points) Consider the voltage divider circuits shown below. The one on the left side is said to be at NO LOAD, while the one on the right is said to be LOADED.



Voltage Divider with no load, $R_L = \infty$



Voltage Divider with load R_L

(a) Show that when the circuit is loaded, $v_{out} = \frac{R_2}{R_1 + R_2 + \frac{R_1 R_2}{R_L}} v_g$.

This was derived in the class.

A test was performed. On no load ($R_L = \infty$), one measured v_{out} as $0.8 v_g$. On the other hand, when $R_L = 12 \text{ K}\Omega$, v_{out} was measured as $0.75 v_g$.

(b) Determine the numerical values of R_1 and R_2 .

On no load ($R_L = \infty$), we have $v_{out} = \frac{R_2}{R_1 + R_2} v_g = 0.8 v_g$. This means

$$\frac{R_2}{R_1 + R_2} = 0.8 \text{ or equivalently } 1 + \frac{R_1}{R_2} = \frac{1}{0.8} = 1.25.$$

With load ($R_L = 12 \text{ K}\Omega$), we have $v_{out} = \frac{R_2}{R_1 + R_2 + \frac{R_1 R_2}{R_L}} v_g = 0.75 v_g$. This means

$$\frac{R_2}{R_1 + R_2 + \frac{R_1 R_2}{12}} = \frac{3}{4} \text{ or equivalently } 1 + \frac{R_1}{R_2} + \frac{R_1}{12} = \frac{4}{3}.$$

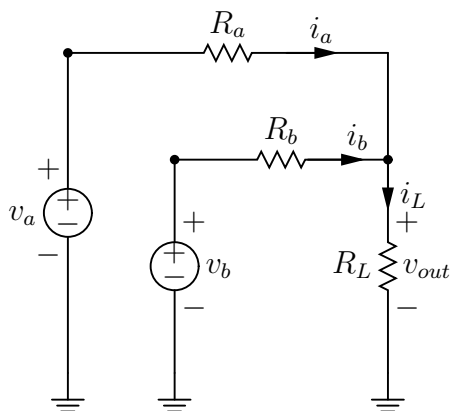
Since $1 + \frac{R_1}{R_2} = 1.25$, we see that $\frac{R_1}{12} = \frac{4}{3} - 1.25$. This gives $R_1 = 1 \text{ K}\Omega$ and then $R_2 = 4 \text{ K}\Omega$.

(c) Let R_1 and R_2 be as determined above. If $v_g = 60\text{V}$, and if accidentally R_L is shorted (that is, R_L is set to zero), determine the power dissipated in R_1 and R_2 .

When R_L is shorted (that is, R_L is set to zero), the entire voltage $v_g = 60\text{V}$ is across R_1 . There exists no voltage across R_2 .

The power dissipated in R_1 is $\frac{60^2}{1\text{K}} = 3.6 \text{ Watts}$.

The power dissipated in R_2 is zero Watts.

Problem 4: (20 points)

Consider the circuit shown on the left where $R_a = 20\Omega$, $R_b = 20\Omega$, and $R_L = 30\Omega$. We would like to solve first for i_L and then for $v_{out} = i_L R_L$ in terms of v_a and v_b . To do so, write down the following laws and then solve for i_L and then for $v_{out} = i_L R_L$.

1. KCL to relate i_a , i_b , and i_L .
2. KVL around the loop containing the source v_a , resistance R_a , and resistance R_L .
3. KVL around the loop containing the source v_b , resistance R_b , and resistance R_L .

Solution: It is easy to write the following three equations:

$$i_a + i_b = i_L \quad \text{KCL} \quad (1)$$

$$v_a - i_a R_a - i_L R_L = v_a - i_a 20 - i_L 30 = 0 \quad \text{KVL} \quad (2)$$

$$v_b - i_b R_b - i_L R_L = v_b - i_b 20 - i_L 30 = 0. \quad \text{KVL} \quad (3)$$

There exists a lot of symmetry in the last two equations. By adding the last two equations and recognizing $i_a + i_b = i_L$, we get

$$v_a + v_b - (i_a + i_b)20 - i_L 60 = 0 \Rightarrow v_a + v_b - i_L 20 - i_L 60 = 0.$$

This implies that

$$i_L = \frac{v_a + v_b}{80} \Rightarrow v_{out} = i_L R_L = \frac{(v_a + v_b)30}{80} = \frac{(v_a + v_b)3}{8}.$$

In general while keeping R_a , R_b , and R_L as symbols, we can solve the equations as follows. We note that there exists a lot of symmetry between the equations (2) and (3). We can exploit such a symmetry. Multiplying equation (2) with R_b and similarly multiplying equation (3) with R_a , we get

$$v_a R_b - i_a R_a R_b - i_L R_L R_b = 0 \quad \text{and} \quad v_b R_a - i_b R_b R_a - i_L R_L R_a = 0.$$

By adding the above two equations and recognizing $i_a + i_b = i_L$, we get

$$v_a R_b + v_b R_a - i_L (R_a R_b + R_a R_L + R_b R_L) = 0.$$

The above equation implies that

$$i_L = \frac{R_b}{R_a R_b + R_a R_L + R_b R_L} v_a + \frac{R_a}{R_a R_b + R_a R_L + R_b R_L} v_b.$$

Then, we immediately get

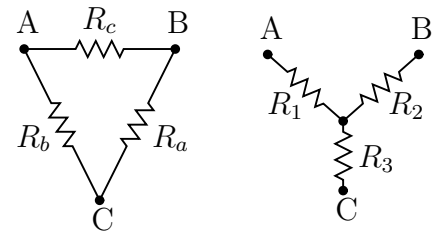
$$v_{out} = i_L R_L = \frac{R_b R_L}{R_a R_b + R_a R_L + R_b R_L} v_a + \frac{R_a R_L}{R_a R_b + R_a R_L + R_b R_L} v_b.$$

For the given values of $R_a = 20\Omega$, $R_b = 20\Omega$, and $R_L = 30\Omega$, we have

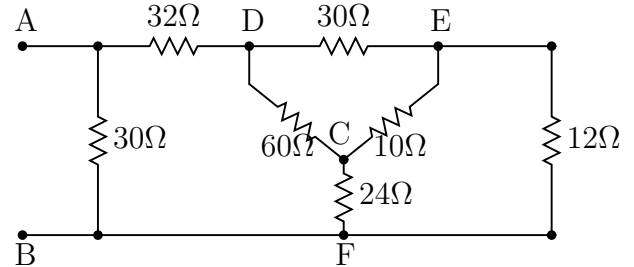
$$v_{out} = \frac{3}{8} v_a + \frac{3}{8} v_b = \frac{(v_a + v_b)3}{8}.$$

The Δ and Y circuit equivalent relationships are as shown:

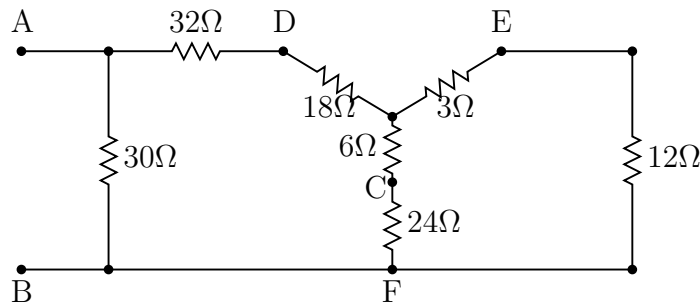
$$\begin{array}{l} R_1 = \frac{R_b R_c}{R_a + R_b + R_c} \\ R_2 = \frac{R_a R_c}{R_a + R_b + R_c} \\ R_3 = \frac{R_a R_b}{R_a + R_b + R_c} \end{array} \quad \left| \quad \begin{array}{l} R_a = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1} \\ R_b = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2} \\ R_c = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3} \end{array} \right.$$



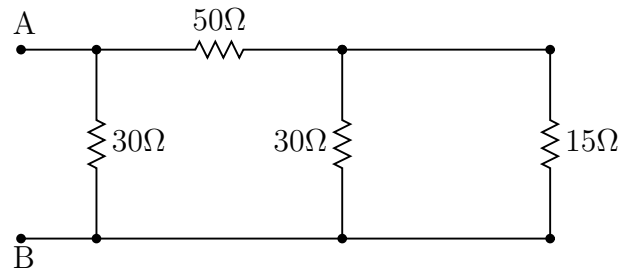
Problem 5: (25 points) Transform the Δ circuit between the terminals C, D, and E to an equivalent Wye circuit. Then, utilizing series and parallel equivalents, determine the resistance seen to the right of terminals A and B. **Draw clearly all the equivalent circuits.**



We can simplify the above circuit by transforming the Δ circuit between the terminals C, D, and E to an equivalent Wye as follows:



By series and parallel equivalents, we can simplify the above circuit sequentially as illustrated below:



It should be relatively easy now to compute the resistance seen to the right of terminals A and B as 20Ω .