332:221 Principles of Electrical Engineering I – Fall 2003
Hourly Exam 2 – November 13, 2003

NAME OF THE STUDENT: Solution
Last four digits of Student ID #: Section # 1 (TTh4) or # 2 (W3,F4)
Circle one

Please make sure that there are 6 pages in this booklet excluding this cover page.

This is a closed-book closed-notes exam. Do all your work on these sheets. If more space is required, ask the instructor.

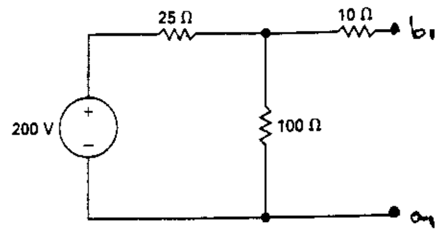
Problem #	Pages	Maximum Points	Points earned	Description
1	1 & 2	30		*Chapter 4, Thevenin equivalent Circuit*
2	3	20		*Chapter 5, Ideal Op – Amp Circuit*
3	4	22		*Chapter 5, Non – Ideal Op – Amp Circuit*
4	5	14		*Chapter 6, Behavior of L*
5	6	14		*Class Notes: Op–Amps, Integrators & Differentiators*

Total points earned by the student =

Note: Most of the problems, if not all, are directly taken from Home-work problems with perhaps numbers changed and slightly modified. My goal in doing so is to encourage you to do Home-work problems. If you simply read the solutions of Home-work problems without doing them, it would not help you.

Curriculum revision: Both the curricula of Electrical option and Computer option have been revised for class year 2006 and onwards. The details are on the ECE website.

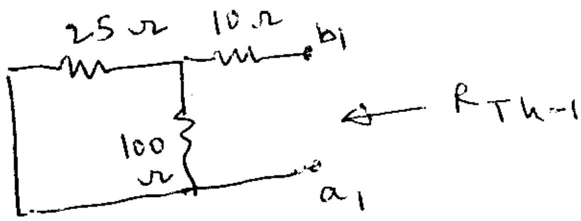
Problem 1: This is HW problem 4.74 re-phrased.
Problem 1a: Construct the Thevenin equivalent circuit with respect to the terminals a_1 and b_1. Let the Thevenin voltage be denoted by $V_{\text{Th-1}}$ and Thevenin resistance by $R_{\text{Th-1}}$.



When there is no load at the terminals a_1 and b_1, there is no current in 10 Ω. Thus open circuit voltage at the terminals a_1 and b_1 is given by (Voltage division rule),

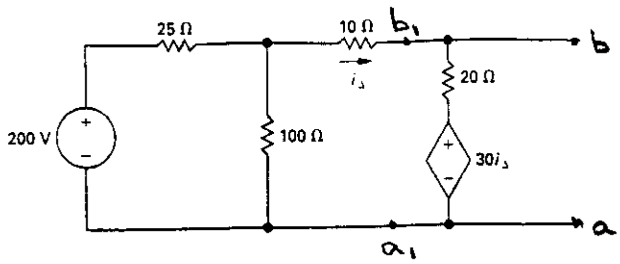
$$V_{OC} = V_{Th-1} = 200 \frac{100}{100+25} = 160\,V$$

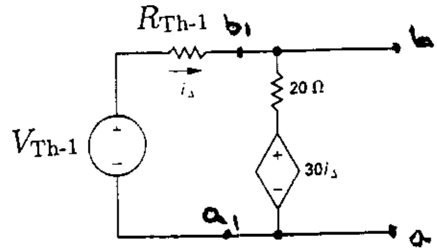
Thevenin resistance is the resistance at terminals a_1 and b_1 when all the independent sources are set to zero.



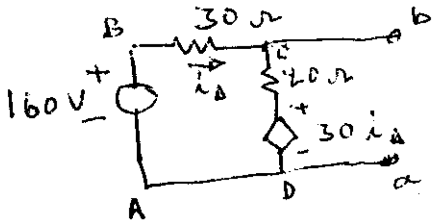
$$R_{Th-1} = 10 + \frac{(100)(25)}{100+25} = 10 + 20 = 30\,\Omega.$$

Problem 1b: We are given the circuit drawn on the left side. However, utilizing the results of **Problem 1a**, we can equivalently redraw it as the one shown on the right side. Construct the Thevenin equivalent circuit with respect to the terminals a and b.





Determination of Open circuit voltage



By writing the KCL for the loop ABCDA, we get

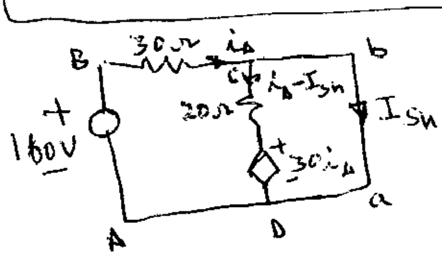
$160 = 30 i_\Delta + 20 i_\Delta + 30 i_\Delta$

$\therefore i_\Delta = 2\text{ A}$

V_{Th} is the voltage across C and D (same as b and a)

$= 20 i_\Delta + 30 i_\Delta = 50 i_\Delta = 100\text{ V}$

Determination of Short circuit current



By mesh analysis,

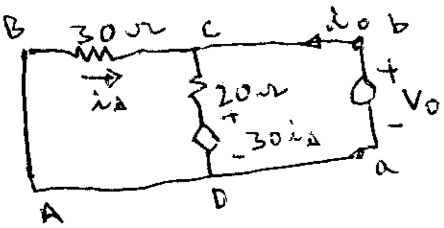
$160 = 30 i_\Delta$ (loop A B C b a D A)

$30 i_\Delta + 20(i_\Delta - I_{sh}) = 0$ (loop D C b a D)

$\therefore i_\Delta = \frac{160}{30}$, $I_{sh} = \frac{50 i_\Delta}{20} = \frac{50}{20}\,\frac{160}{30} = \frac{40}{3}\text{ A}$

Thevenin resistance $= \frac{V_{oc}}{I_{sh}} = \frac{100 \times 3}{40} = 7.5\ \Omega$

Determination of Thevenin resistance by looking for the resistance at terminals a and b when all the independent sources are set to zero!



Let $i_o = 1\text{ A}$.

$V_o = -30 i_\Delta$ (loop A B C b a D A)

$\therefore i_\Delta = -\frac{V_o}{30}$

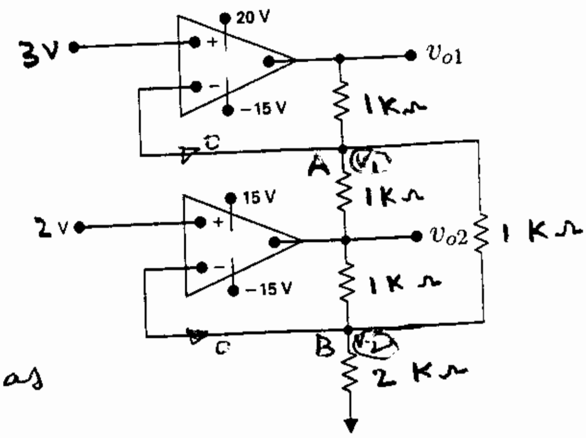
$V_o = 20(1 + i_\Delta) + 30 i_\Delta$ (loop D C b a D)

$= 50\left(\frac{-V_o}{30}\right) + 20$

$\therefore V_o = \frac{(20)(30)}{80} = 7.5\text{ V}$

Thevenin resistance $= \frac{V_o}{i_o} = \frac{V_o}{1} = 7.5\ \Omega$

Problem 2: This is HW problem 5.33 re-phrased. Assuming that the Op-Amps are ideal, determine v_{o1} and v_{o2}.



As usual we take the ground as the reference node.

V_1 = Voltage at A = 3 V (Input terminals of TOP OP-Amp are Virtual short)

V_2 = Voltage at B = 2V (Input terminals of lower OP-Amp are Virtual short).

Node equation at A :

$$\frac{3-V_{o1}}{1K} + \frac{3-V_{o2}}{1K} + \frac{3-2}{1K} = 0$$

$$\therefore 7 - V_{o1} - V_{o2} = 0$$

Node equation at B ;

$$\frac{2-0}{2K} + \frac{2-V_{o2}}{1K} + \frac{2-3}{1K} = 0$$

$$\boxed{\therefore V_{o2} = 2\ V}$$

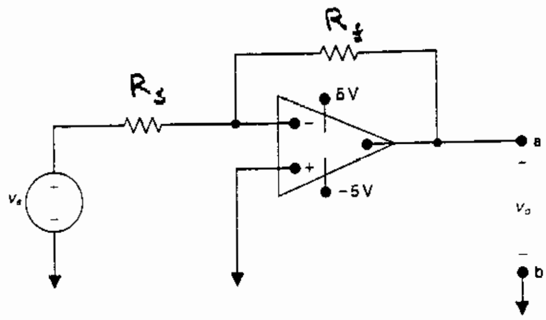
$$7 - V_{o1} - 2 = 0$$

$$\boxed{\therefore V_{o1} = 5\ V}$$

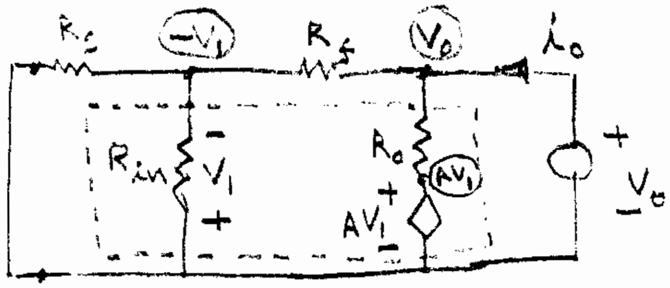
Problem 3: This is HW problem 5.43 re-phrased. The Op-Amp in the given circuit is non-ideal with its input resistance $R_{in} = 500\,K\Omega$, its output resistance $R_0 = 0.5\,K\Omega$, and its gain $A = 10^5$. Also, let $R_s = 50\,K\Omega$, and $R_f = 50\,K\Omega$. Determine the output resistance R_{out} of the circuit. An answer of $R_{out} = 0$ is not acceptable. Atleast one significant bit of non-zero value of R_{out} should be indicated.

Be careful with your algebra.

A term can be neglected only if it is small compared to another term which is added or subtracted from it.



By using a model for OP-Amp, the output resistance R_{out} is given by the ratio $\frac{V_O}{i_O}$ in the following circuit:



We will use the nodal analysis. Nodes are marked as shown.

$$\frac{-V_1}{50K} - \frac{V_1}{500K} - \frac{V_1+V_O}{50K} = 0$$

$$-2.1V_1 - V_O = 0 \quad \therefore \; V_1 = -\frac{1}{2.1}V_O$$

$$\frac{V_O+V_1}{50K} + \frac{V_O-10^5V_1}{0.5K} = i_O$$

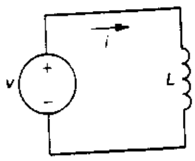
Substituting for $V_1 = -\frac{1}{2.1}V_O$,

$$V_O\left[\frac{1}{50K} + \frac{1}{0.5K} - \frac{1}{(2.1)50K} + \frac{10^5}{(2.1)(0.5)K}\right] = i_O$$

$$V_O\left[\underbrace{(0.2)10^{-4} + 2(10^{-3}) - (0.476)10^{-4}}_{\text{Negligible in comparision with } (0.952)10^2} + (0.952)10^{2}\right] = i_O$$

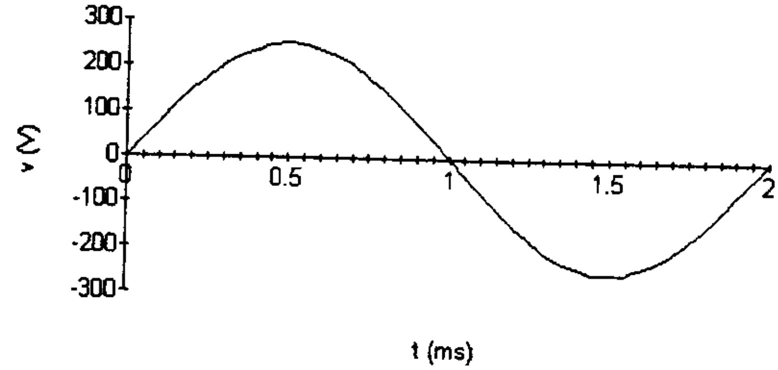
$$\therefore \; R_{out} = \frac{V_O}{i_O} \simeq \frac{1}{(0.952)10^2} \simeq (1.05)10^{-2} \simeq 0.0105\,\Omega$$

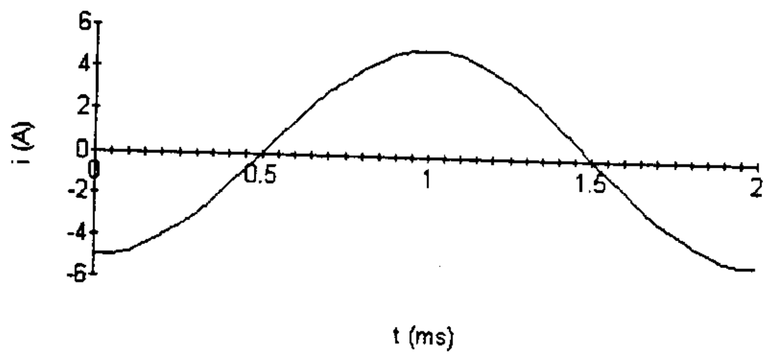
Problem 4: This is HW problem 6.10 re-phrased. Consider the circuit on the right in which $v = 250\sin(1000t)$ V, $L = 50$ mH, and $i(0) = -5$ A. Find an expression for the inductor current $i(t)$. Also, sketch v, i, power consumed p, and energy stored w in the inductance L with respect to time t. Label the axes appropriately and show atleast one cycle of each variable on your sketch.

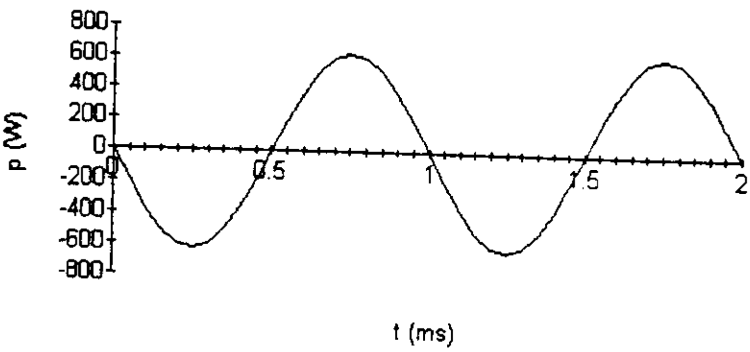


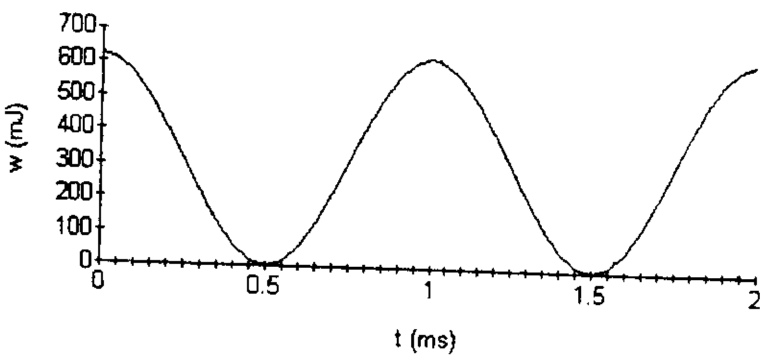
$$\begin{aligned}
i &= \frac{1000}{50}\int_0^t 250\sin 1000x\,dx - 5 \\
&= 5000\int_0^t \sin 1000x\,dx - 5 \\
&= 5000\left[\frac{-\cos 1000x}{1000}\right|_0^t - 5 \\
&= 5(1-\cos 1000t) - 5 \\
i &= -5\cos 1000t \text{ A}
\end{aligned}$$

$$\begin{aligned}
p &= vi = (250\sin 1000t)(-5\cos 1000t) \\
&= -1250\sin 1000t\cos 1000t \\
p &= -625\sin 2000t \text{ W} \\
w &= \frac{1}{2}Li^2 \\
&= \frac{1}{2}(50\times 10^{-3})25\cos^2 1000t \\
&= 625\cos^2 1000t \text{ mJ} \\
w &= [312.5 + 312.5\cos 2000t] \text{ mJ.}
\end{aligned}$$



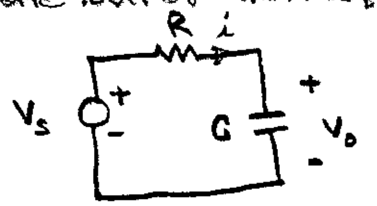






Problem 5: Consider the circuits shown. Explain how and when each circuit acts as a differentiator or an integrator. If proper explanation is lacking, **no credit** will be given.

The circuit is approximately are integration if $RC \gg 1$.



$$i = C\frac{dV_o}{dt}$$

$$V_s = Ri + V_o = RC\frac{dV_o}{dt} + V_o$$

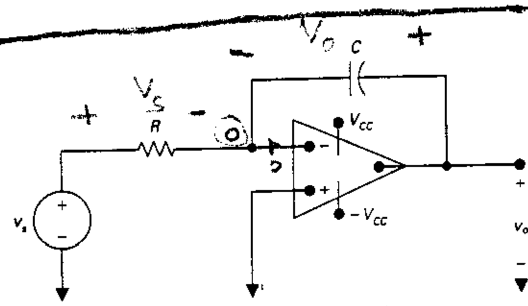
If RC is large, then

$$V_s \simeq RC\frac{dV_o}{dt} \quad \therefore \frac{dV_o}{dt} = \frac{1}{RC}V_s$$

$$V_o(t) = V_o(0) + \frac{1}{RC}\int_0^t V_s(t)\,dt$$

Assume an ideal OP-AMP.

Node equation at the negative input terminal:



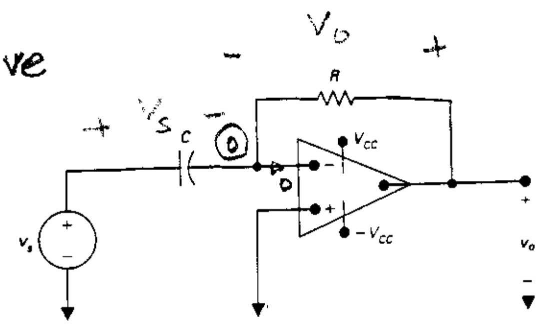
$$-\frac{V_s}{R} - C\frac{dV_o}{dt} = 0$$

$$\therefore \frac{dV_o}{dt} = -\frac{1}{RC}V_s \qquad \text{Hence } V_o(t) = V_o(t) - \frac{1}{RC}\int_0^t V_s(t)\,dt$$

It is an integrator circuit.

Assume an ideal OP-AMP

Node equation at the negative input terminal:



$$-C\frac{dV_s}{dt} - \frac{V_o}{R} = 0$$

$$\therefore V_o = -RC\frac{dV_s}{dt}$$

It is a differentiator circuit.