THE STATE UNIVERSITY OF NEW JERSEY
RUTGERS

School of Engineering
Department of Electrical and Computer Engineering

332:223 Principles of Electrical Engineering I Laboratory

Experiment V

# *Sinusoidal Steady-State Analysis*

## 1 Introduction

**Objectives**

- To demonstrate the properties of simple series R-C, R-L and R-L-C circuits.
- To demonstrate phasor analysis as a tool for the analysis of circuits containing frequency-dependent impedances.
- To experimentally explore the effect of Capacitors and Inductors on the phase of voltages and currents in a circuit

**Overview**

This experiment is designed to demonstrate the analysis of R-C, R-L and R-L-C circuits by the use of phasors. A summary of phasors and phasor analysis of sinusoidal steady state is presented in section 2.
The prelab exercises are designed to promote familiarity with the concepts and involve calculations of the values of several elements that will be used in the experiments.
The three actual laboratory experiments are designed to verify the concepts by direct measurement of voltages, currents and resistances. The difference of a real and an ideal inductor is made clear with the measurement of the associated d.c. resistance and its incorporation in the calculation of the resistance value to be used experimentally.


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# 2 Theory

## 2.1 The sinusoidal source[1]

A sinusoidal voltage or current source (independent or dependent) produces a voltage or current that varies sinusoidally with time. A sine wave (say an alternating periodic voltage v(t)) is uniquely described by the equation

$$v(\mathrm{t}) = V_m \cos(\omega t + \phi) + V_{avg}$$

As mentioned in lab II, all of the above parameters (*amplitude* $V_m$, *frequency* $\omega$, *phase angle* $\phi$ *and DC offset* $V_{avg}$) are needed to uniquely define a signal. For this reason each of them is called a *partial descriptor*. If the DC offset $V_{avg}$ is of no importance, only the ac quantity is dealt with:

$$v(\mathrm{t}) = V_m \cos(\omega t + \phi) \qquad (1)$$

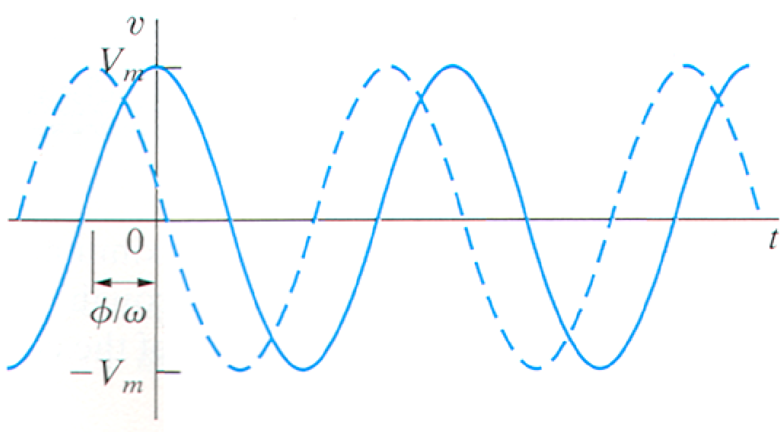



Fig. 1 Sinusoidal voltages with and without phase shift

- Since the cosine function is bounded by ±1, the amplitude is bounded by $\pm V_m$.
- The phase angle $\phi$ describes the phase shift of the *waveform compared with a time origin*. Changing $\phi$ shifts the sinusoidal function along the time axis, but has no effect on either the amplitude $V_m$ or the angular frequency $\omega$.
- If $\phi>0$, the sinusoidal function shifts to the left; if $\phi<0$, it shifts to the right.
- The *angular frequency* $\omega$ has units of radians/sec. It is related to the *frequency* of the wave, $f$ (Hz) by:

$$\omega = 2\pi f$$

[1] A more detailed description can be found in section 9.1 of the text.

## 2.2 Phasor representation of a sinusoid [2]

Using the Euler identity, $e^{\pm j\theta} = \cos(\theta) \pm j\sin(\theta)$

Eq. 1 can be written as follows:

$$v(t) = V_m \Re\{e^{j(\omega t + \phi)}\} = V_m \Re\{e^{j\omega t} e^{j\phi}\} = \Re\{(V_m e^{j\phi})e^{j\omega t}\}$$

The complex number $V_m e^{j\phi}$ is called the *phasor representation* or the *phasor transform* of *v(t)*. Phasors include only two of the partial descriptors e.g. they convey no frequency information but they greatly simplify notation. For example Eq. 1 can be represented in the phasor domain as follows[3]:

$$\boldsymbol{V} = V_m \angle\phi$$

## 2.3 The passive circuit elements in the Phasor Domain [4]

If *i(t)* is the current flowing through a resistor, inductor, or a capacitor, and

$i(t) = I_m \cos(\omega t + \theta_i)$ then in phasor notation $\boldsymbol{I} = I_m \angle \theta_i$

and the voltage in the phasor domain in each case is:

a- Across a resistor:

$$\boldsymbol{V} = R\boldsymbol{I} = RI_m \angle \theta_i \tag{2}$$

b- Across an inductor:

$$\boldsymbol{V} = (j\omega L)\boldsymbol{I} = \omega L I_m \angle \theta_i + 90^\circ \tag{3}$$

c- Across a capacitor:

$$\boldsymbol{V} = (1/j\omega C)\boldsymbol{I} = -(j\omega C)\boldsymbol{I} = I_m/\omega C \angle \theta_i - 90^\circ \tag{4}$$

[2] A more detailed description can be found in section 9.3 of the text.
[3] In agreement with the notation of the text, phasors and only phasors will be represented by capital bold symbols throughout this lab.
[4] The subject is treated in more detail in section 9.4 of the text.

## 2.4 Phasor Diagrams [5]

Consider a series RLC circuit with a sinusoidal voltage input $\boldsymbol{V_s}$ as shown in fig. 2.

$$v_s(\mathrm{t}) = V_{sm} \cos(\omega t + \phi)$$

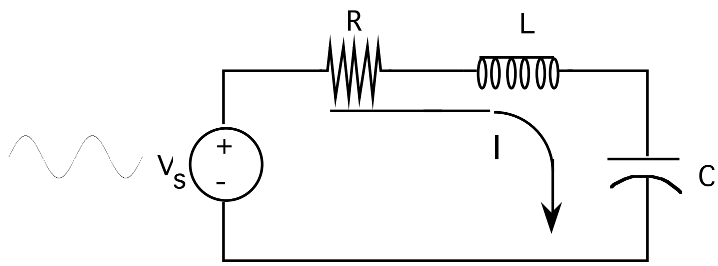


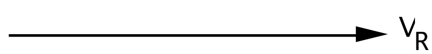

Fig. 2
A Series RLC Circuit

In the phasor domain, $\boldsymbol{V_s} = \boldsymbol{V_R} + \boldsymbol{V_L} + \boldsymbol{V_C} = R\boldsymbol{I} + j\omega L\boldsymbol{I} - j\omega C\boldsymbol{I}$

$$= RI_m \angle \theta_i + \omega LI_m \angle \theta_i + 90^o + I_m/\omega C \angle \theta_i - 90^o$$

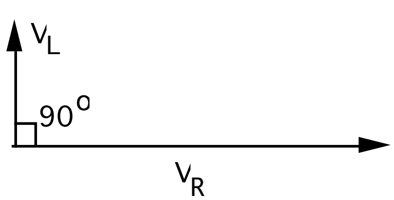
To draw the *phasor diagram* of Fig. 2, for ideal circuit elements, one needs to proceed as follows:

(1) First a *reference voltage* is chosen, say $\boldsymbol{V_R}$; in other words the angle $\theta_{VR}$ is set to zero; since there is no phase difference between $\boldsymbol{V_R}$ and $\boldsymbol{I}$, this means $\theta_i = \theta_{VR} = 0$.




(a)

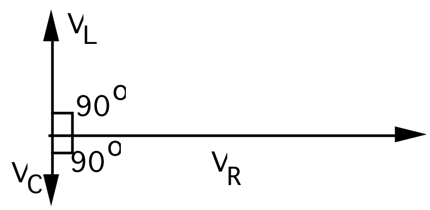
(2) From the equation above it is clear that $\boldsymbol{V_L} = \omega LI_m \angle 90^o$ will lead $\boldsymbol{V_R}$ by 90°




(b)

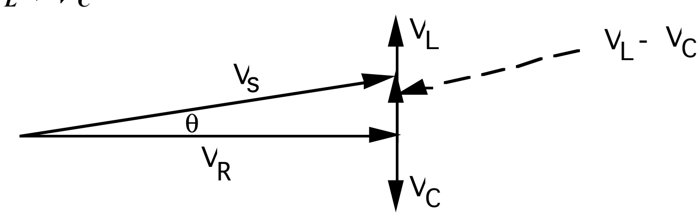
[5] The subject is treated in more detail in section 9.12 of the text.

(3) From the equation above it is clear that $\boldsymbol{V_C} = I_m/\omega C \angle{-90^o}$ will lag behind $\boldsymbol{V_R}$ by $90^o$




(c)

(4) $\boldsymbol{V_s} = \boldsymbol{V_R} + \boldsymbol{V_L} + \boldsymbol{V_C}$




(d)

From inspection of the resulting triangle it follows that:

- The magnitude of $\boldsymbol{V_s}$ is: $|\boldsymbol{V_s}| = [V_R^2 + (V_L - V_C)^2]^{1/2}$
- The phase angle of $\boldsymbol{V_s}$ is: $\theta = \tan^{-1}[(V_L - V_C)/V_R]$

# 3 Prelab Exercises

## 3.1 R-C Branch

Refer to Fig.3 and compute R and C for $Z = 2556\angle\underline{-38.5^{o}}\ \Omega$ at f = 1000Hz. Note that

$$Z = R - \frac{j}{\omega C} = \sqrt{R^2 + \frac{1}{\omega^2 C^2}}\ \angle - \tan^{-1}\left(\frac{1}{\omega CR}\right)$$

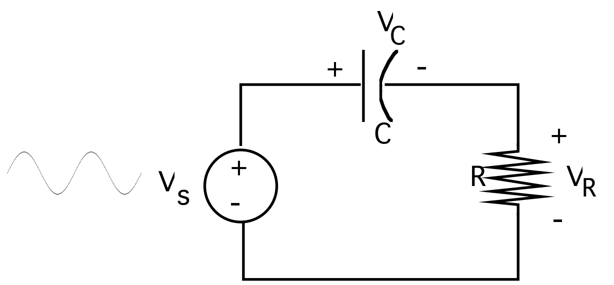
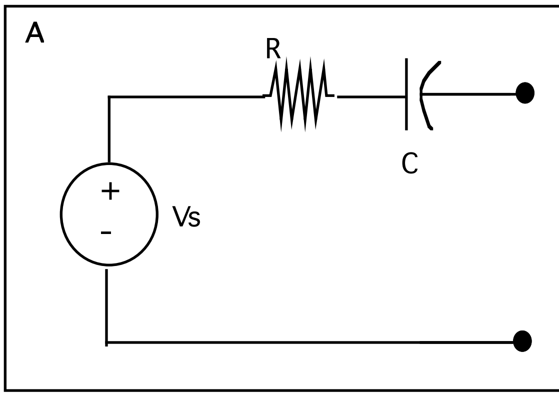



Fig. 3
R-C Branch

## 3.2 Maximum power A

For the following source circuit, design a load circuit that will extract maximum power from it. Your design should minimize the number of parts and must be within 5% of transferring maximum power to the load.
Calculate the complex power that is delivered to the load.




Source Circuit Specifications

$f$ = 5 kHz
$\mathbf{V}_s = 5\angle\underline{0^{o}}$ V
$R$ = 470 Ω
$C$ = 0.15 μF

## 3.3 R-L Branch

Refer to Fig. 4 and compute R and L for Z = $1181\angle 32.14^{\circ}\ \Omega$ at f=1000Hz. Note that

$$Z = R + j\omega L = \sqrt{R^2 + \omega^2 L^2}\ \angle \tan^{-1}(\omega L / R)$$

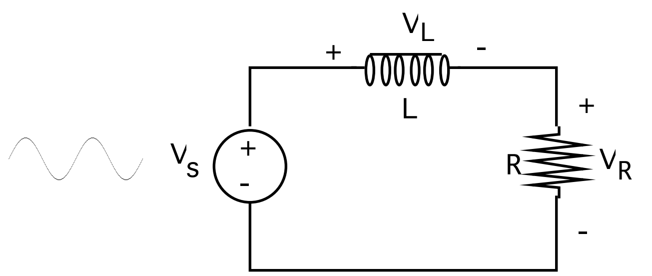
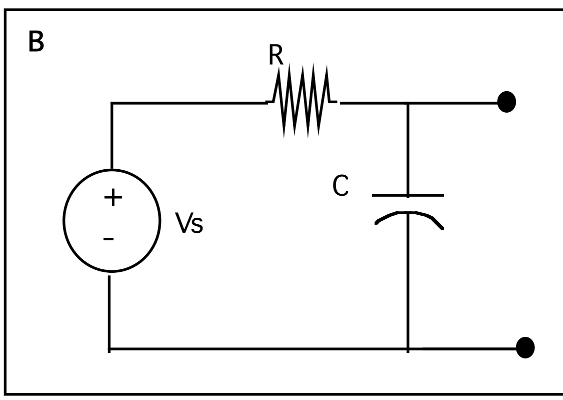



Fig. 4
R-L Branch

## 3.4 R-C branch maximum power B

For the following source circuit, design a load circuit that will extract maximum power from it. Your design should minimize the number of parts and must be within 5% of transferring maximum power to the load.
Calculate the complex power that is delivered to the load.




| Source Circuit Specifications |
|---|
| $f$ = 10 kHz<br>$V_s$= $5\angle 0^{\circ}$ V<br>$R$ = 10 KΩ<br>$C$= 0.015 μF |

### 3.5 R-L-C branch

Refer to Fig. 5 and compute R and C for Z = 1388 $\angle\underline{-43.9^{o}}$ Ω at f = 1000Hz, given L = 100mH. Note that

$$Z = R + j\left(\omega L - \frac{1}{\omega C}\right) = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}\ \angle \tan^{-1}\left(\frac{\omega L - \frac{1}{\omega C}}{R}\right)$$

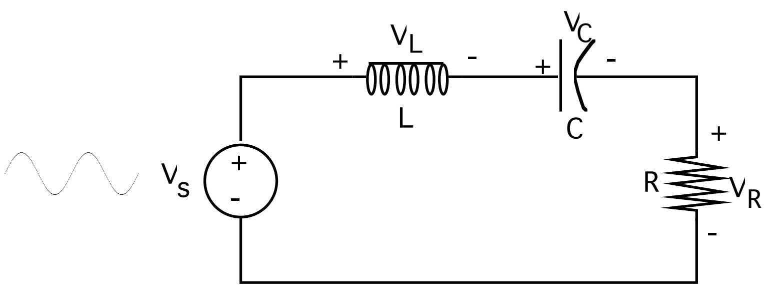



Fig. 5
R-L-C Branch

# 4 Experiments

Suggested Equipment:

Tektronix FG 501A 2MHz Function Generator[6]
Tektronix DC 504A Counter-Timer
Protek B-845 Digital Multimeter
Agilent 54622A oscilloscope
LS-400A Inductance Substitute Box
1 μF Capacitor
100 KΩ resistor
Various circuit elements to be determined by students

## 4.1 R-C branch

a. With the values of R and C as computed in pre-lab exercise 3.1, build the circuit of Fig.3.

b. Connect the DVM (switched to AC) across the resistor and adjust the oscillator[7] output[8] until $V_R$ has the proper value that corresponds to a current of 1mA flowing through the circuit.

c. Measure $V_s$, $V_R$, and $V_C$.

d. Display the time domain signals $v_s(t)$ and $v_R(t)$ corresponding to $\boldsymbol{V_s}$ and $\boldsymbol{V_R}$ on the oscilloscope, and download them to your floppy disk.

e. Measure the phase. The oscilloscope can automatically calculate the phase difference angle θ between the phasors $\boldsymbol{V_s}$ and $\boldsymbol{V_R}$ in the following way:
   - press the *Quick Meas* button
   - press the 2nd soft key to select phase
   - press the 3rd soft key to measure the phase difference between the two signals

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[6] Throughout this experiment, large resistance values will be used; thus, the 50Ω internal impedance of the function generator can be neglected.

[7] "Oscillator" is routinely used for any source of ac voltage (here the Function Generator).

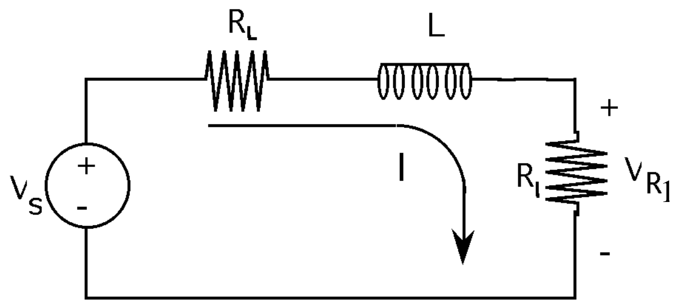
[8] The oscillator output and the oscilloscope input have a common ground connection, consequently a component voltage cannot be displayed properly on the scope unless that component is also connected to the common ground. Make sure that you build your circuits exactly as shown in the figures.

## 4.2 R-L Branch

a. Select an inductor which has the inductance value L computed in pre-lab exercise 3.3.

b. Measure the d.c. resistance $R_L$ of the inductor using the digital ohmmeter.

c. In pre-lab exercise 3.3, a value for the resistance R of Fig.4 was computed. This computation, however, assumed that L was a *pure inductance*. It is now seen that a real inductor possesses resistance ($R_L$) *in addition* to its inductance (L). In other words the actual circuit looks like




Since R must be the total resistance in the circuit (as per 3.3 calculations) the resistance of the inductor has to be taken into account. Let then

$$R_1 = R - R_L$$

where $R_L$ is the d.c. resistance of the inductor.
Select a resistor whose resistance value equals $R_1$. In other words, the resistance value of the resistor is intentionally decreased in order to take the d.c. resistance of the inductor into account.

d. With the values of $R_1$ and L as described above, build the series circuit of Fig.4. Set up the circuit exactly as shown in Fig. 4 in order to avoid problems with ground.

e. Adjust the oscillator until the output $V_{R1}$ across the resistor $R_1$ has the proper value that corresponds to a 1mA current flowing through the circuit.

f. Measure the magnitudes of the voltages corresponding to $\boldsymbol{V_s}$, $\boldsymbol{V_{R1}}$, and $\boldsymbol{V_{L+RL}}$ using the DVM (switched to AC). Note that the physical inductor is a series combination of $R_L$ and L. One cannot physically measure the inductance voltage alone. Only the voltage across the inductor which has both the resistance $R_L$ and inductance L can be measured.

g. Display the time domain signals $v_s(t)$ and $v_{R1}(t)$ corresponding to $\boldsymbol{V_s}$ and $\boldsymbol{V_{R1}}$ on the oscilloscope, and download them to your floppy disk.

h. Measure the phase angle θ between $\boldsymbol{V_s}$ and $\boldsymbol{V_{R1}}$ using the method explained in Section 4.1.e.

## 4.3 R-L-C Branch

a. Measure the d.c. resistance $R_L$ of the inductor whose inductance value is L = 100mH.

b. Build the R-L-C series circuit of Fig. 5 by using the R and C values as computed in pre-lab exercise 3.5. Make sure that the value of resistance R is decreased to $R_1 = R - R_L$ by the d.c. resistance of the inductor as in 4.2.c.

c. Follow the steps in Section 4.1 and measure $V_s$, $V_{R1}$, $V_{L+RL}$, $V_C$, and the phase angle θ between $\boldsymbol{V_s}$ and $\boldsymbol{V_{R1}}$; download the waveforms $v_s(t)$ and $v_{R1}(t)$ corresponding to $\boldsymbol{V_s}$ and $\boldsymbol{V_{R1}}$ from the oscilloscope.

d. At resonance the reactance X is zero and the impedance is purely resistive. Since the imaginary portion of the impedance is zero the phase angle is equal to zero.
Measure the resonant frequency for the circuit of Fig. 5. There are a number of ways of doing so. One way is to tune the frequency of the oscillator until $\boldsymbol{V_s}$ and $\boldsymbol{V_{R1}}$ are in phase, (i.e. the phase angle between them is zero) by watching $v_s(t)$ and $v_{R1}(t)$.

e. For all frequencies above resonance, phase angles have the same sign and for frequencies below the resonant point they have the opposite sign. Verify that this is true over a frequency range of two orders of magnitude extending on both sides of the resonant frequency.

# 5 Report

5.1 For the R-C branch shown in Fig.3, do the following:

a. Tabulate $V_s$, $V_R$, $V_C$, and θ measured in the experiment.

b. Submit a copy of the waveforms $v_s(t)$ and $v_R(t)$ of $\boldsymbol{V_s}$ and $\boldsymbol{V_R}$

c. Prepare a phasor diagram to scale that represents in phasor domain

$$\boldsymbol{V_s} = \boldsymbol{V_R} + \boldsymbol{V_C}.$$

d. Calculate the phase angle θ in three different ways:

$$\theta = -\tan^{-1}(V_C/V_R) = -\sin^{-1}(V_C/V_s) = -\cos^{-1}(V_R/V_s)$$

Compare θ calculated with θ measured.

e. Calculate the average power[9], P, in two different ways:

$$\mathrm{P} = V_R^{\,2}/R = V_s I \cos\theta$$

where θ is the measured phase angle.

f. Does $\boldsymbol{V_R}$ lead or lag $\boldsymbol{V_s}$?

g. For the circuit of Fig. A in pre-lab exercise 3.2, what is the load that consumes the maximum power and what is that value of maximum power?

5.2 For the R-L branch shown in Fig.4, do the following:

a. Tabulate $V_s$, $V_{R1}$, $V_{L+RL}$, and θ measured.

b. Submit a copy of the waveforms $v_s(t)$ and $v_{R1}(t)$ of $\boldsymbol{V_{R1}}$ and $\boldsymbol{V_s}$.

c. Prepare a phasor diagram to scale. Take the effect of the d.c. resistance of the inductor into account.

d. From the phasor diagram, determine the phase angle θ in three different ways as in item 5.1.d. Compare with the measured one.

e. Calculate the average power, P, in two different ways as in 5.1.e.

f. Does $\boldsymbol{V_R}$ lead or lag $\boldsymbol{V_s}$?

h. For the circuit of Fig. B in pre-lab exercise 3.4, what is the load that consumes the maximum power and what is that value of maximum power.

5.3 For the R-L-C branch in Fg. 5, repeat 5.2 (a-f) for the impedance Z used in performing the experiment. Do not forget to take into account the d.c. resistance $R_L$ of the inductor L in all calculations.

5.4 With $v_s(t) = 10\sin(2\pi f t)$ Volts where $f$=1KHz, use PSpice to plot $v_s(t)$ and $v_R(t)$ for the three circuits in figures 3, 4, and 5. Show 2 to 4 periods of the waveforms in your plot. (Hint: Use the TRAN. command.)

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[9] For detailed discussion on average power see section 10.2 of the text.

5.5 With $v_s(t) = 10\sin(2\pi ft)$ Volts where $f$=1KHz, use PSpice to determine the real part, imaginary part, magnitude, and phase of $V_R$ (or $V_{R1}$ as the case may be) for the circuits in figures 3, 4, and 5. ( Hint: Use the AC. command.)

5.6 Prepare a summary.