2. This problem illustrates transient and steady-state sinusoidal responses. Consider a signal consisting of three sinusoidal bursts (shown at end):

\[
\begin{cases}
\sin(3t), & 0 \leq t < 30 \\
\sin(2t), & 30 \leq t < 70 \\
\sin(3t), & 70 \leq t < 100
\end{cases}
\]

It can be generated over a period \(0 \leq t \leq 100\) by the code:

```matlab
Tmax = 100; T = Tmax/1000; t = 0:T:Tmax;
f = sin(3*t) .* upulse(t,30) + ... sin(2*t) .* upulse(t-30,40) + ... sin(3*t) .* upulse(t-70,30);
```

It is desired to eliminate the middle burst by means of a notch filter:

\[
H(s) = \frac{s^2 + \omega_0^2}{s^2 + \alpha s + \omega_0^2}
\]

where \(\omega_0 = 2\) is the notch frequency coinciding with the frequency of the middle burst, and \(\alpha = 0.3\) is a parameter that represents the 3-dB width of the notch (see graph on last page). As discussed in class, the impulse response of this filter is:

\[
h(t) = \delta(t) - g(t), \quad g(t) = \alpha e^{-\alpha t/2} \left[ \cos(\omega_0 t) - \frac{\alpha}{2\omega_r} \sin(\omega_0 t) \right] u(t)
\]

where \(\omega_r = \sqrt{\omega_0^2 - \alpha^2/4}\). It follows that the output signal will be:

\[
y(t) = \int h(t-\tau) f(\tau) d\tau = f(t) - \int g(t-\tau) f(\tau) d\tau
\]

which can be implemented by the MATLAB code:

```matlab
y = f - T * conv(g,f);
```

(a) Compute the above output signal \(y(t)\) and plot it versus \(t\). On a separate graph, but using the same vertical and horizontal scales, plot the input signal \(f(t)\). Note the removal of the middle burst after the transients have decayed. Explain quantitatively the slight attenuation of the first and third bursts. Repeat with \(\alpha = 0.1\), which corresponds to a narrower notch, but with a longer time constant.

(b) For the case \(\alpha = 0.3\), plot \(h(t)\) versus \(0 < t < T_{\text{max}}\). The time constant of the filter is the effective duration of \(h(t)\). On another graph, plot the magnitude response \(|H(\omega)|^2\) over \(0 \leq \omega \leq 5\).
Typical Outputs

Convolution, $N = 101$

Input signal, $f(t)$

Output signal, $y(t)$

Notch filter magnitude response, $|H(\omega)|^2$

Notch filter impulse response