Outline

Slotted CSMA

Stabilized Slotted CSMA

Unslotted CSMA

Reference

D. Bertsekas and R. Gallager, "Data networks."

CSMA

$\beta \equiv$ Propagation and detection delay in packet transmission units

$\tau \equiv$ Delay in seconds

$C \equiv$ Raw channel bit rate

$L \equiv$ Expected number of bits in a data packet

$$\Rightarrow \beta = \frac{\tau \cdot C}{L} \tag{4.33}$$

If nodes detect idle periods quickly $\Rightarrow$ terminate idle period and allow nodes to initiate packet transmissions.

Assume (0,1,e) feedback with delay β

Assume further infinite nodes and Poisson arrivals with rate λ

CSMA Slotted Aloha

Nonpersistent CSMA:

Packet arrival is regarded as backlogged if it arrives while a transmission is in progress

$\Rightarrow$ Retransmit in next idle slot with probability q_r

Time between state transitions (including self-transitions) is either β (for an idle slot) or $1+\beta$ (for a busy slot)

$$P_{\text{idle}} = e^{-\lambda \cdot \beta} \cdot (1-q_r)^n$$

$$P_{\text{busy}} = 1 - P_{\text{idle}} = 1 - e^{-\lambda \cdot \beta} \cdot (1-q_r)^n$$

$$E[\text{time between state transitions}] = \beta \cdot P_{\text{idle}} + (1+\beta) \cdot P_{\text{busy}}$$
$$\beta \cdot P_{\text{idle}} + (1+\beta) \cdot (1 - P_{\text{idle}})$$
$$= \beta \cdot P_{\text{idle}} + 1 + \beta - P_{\text{idle}} - \beta \cdot P_{\text{idle}} = 1 + \beta - P_{\text{idle}}$$
$$= 1 + \beta - e^{-\lambda \cdot \beta} \cdot (1-q_r)^n$$

Thus, the expected number of arrivals between state transitions is simply:

$$E[\text{arrivals}] = \lambda \cdot \left[1 + \beta - e^{-\lambda \cdot \beta} \cdot (1-q_r)^n\right] \qquad (4.34)$$

The expected number of departures between state transitions in state n is simply the probability of a successful transmission

$$P_{\text{success}} = Q_a(1) \cdot Q_r(0,n) + Q_a(0) \cdot Q_r(1,n)$$

$Q_a(\cdot) \sim$ Poisson with parameter $\lambda \cdot \beta$

$Q_r(\cdot, n) \sim$ Binomial with parameter q_r

$$\Rightarrow P_{\text{success}} = \lambda \cdot \beta \cdot e^{-\lambda \cdot \beta} \cdot (1-q_r)^n + e^{-\lambda \cdot \beta} \cdot n \cdot q_r \cdot (1-q_r)^{n-1}$$

$$\Rightarrow P_{\text{success}} = \left(\lambda \cdot \beta + \frac{n \cdot q_r}{1-q_r} \right) \cdot e^{-\lambda \cdot \beta} \cdot (1-q_r)^n \qquad (4.35)$$

$D_n \equiv$ Expected number of arrivals less the expected number of departures between state transitions

$$D_n = E[\text{arrivals}] - P_{\text{success}}$$

$$\Rightarrow D_n = \lambda \cdot \left[1 + \beta - e^{-\lambda \cdot \beta} \cdot (1-q_r)^n \right] - \left(\lambda \cdot \beta + \frac{q_r \cdot n}{1-q_r} \right) \cdot e^{-\lambda \cdot \beta} \cdot (1-q_r)^n$$

$$(4.36)$$

Applying the approximations $(1-q_r)^{n-1} \approx (1-q_r)^n \approx e^{-q_r \cdot n}$ and $1-q_r \approx 1$ for small q_r:

$$\Rightarrow D_n \approx \lambda \cdot \left(1 + \beta - e^{-\lambda \cdot \beta - q_r \cdot n}\right) - (\lambda \cdot \beta + q_r \cdot n) \cdot e^{-\lambda \cdot \beta - q_r \cdot n}$$

$g(n) \equiv$ Expected number of packet transmissions in the next idle slot (β time units) following transition to state n

$$\Rightarrow g(n) = \lambda \cdot \beta + q_r \cdot n \tag{4.38}$$

Applying $g(n)$ to the above expression for drift yields:

$$D_n = \lambda \cdot \left(1 + \beta - e^{-g(n)}\right) - g(n) \cdot^{-g(n)} \tag{4.37}$$

Clearly, D_n is negative if:

$$\lambda < \frac{g(n) \cdot e^{-g(n)}}{1 + \beta - e^{-g(n)}} \tag{4.39}$$

The numerator is the expected number of departures per state transition and the denominator is the expected duration of a state transition period.

Thus, the ratio of (4.39) is also the number of departures per unit time, i.e., the throughput:

$$\Rightarrow E[\text{throughput}] = \frac{g(n) \cdot e^{-g(n)}}{1 + \beta - e^{-g(n)}}$$

E[throughput] maximized at $g(n) = \sqrt{2 \cdot \beta}$

$g(n) = \sqrt{2 \cdot \beta}$???

$$\frac{d}{dg(n)}\left\{\frac{g(n)\cdot e^{-g(n)}}{1+\beta-e^{-g(n)}}\right\}=0$$

Letting $g = g(n)$:

$$\Rightarrow \frac{d}{dg}\left\{\frac{g\cdot e^{-g}}{1+\beta-e^{-g}}\right\}=0$$

$$\Rightarrow \frac{\left(1+\beta-e^{-g}\right)\cdot\left(e^{-g}-g\cdot e^{-g}\right)-g\cdot e^{-g}\cdot\left(e^{-g}\right)}{1+\beta-e^{-g}}=0$$

$$\Rightarrow \left(1+\beta-e^{-g}\right)\cdot(1-g)-g\cdot e^{-g}=0$$

$$\Rightarrow 1+\beta-e^{-g}-(1+\beta)\cdot g+g\cdot e^{-g}-g\cdot e^{-g}=0$$

$$\Rightarrow 1+\beta-(1+\beta)\cdot g-e^{-g}=0$$

Applying the approximation $e^{-g} \approx 1-g+g^2/2$:

$$\Rightarrow 1+\beta-(1+\beta)\cdot g-\left(1-g+g^2/2\right)=0$$

$$\Rightarrow \beta-\beta\cdot g-g^2/2=0$$

$$\Rightarrow g^2/2-\beta\cdot g+\beta=0 \qquad (***)$$

Applying the Quadratic Formula yields:

$$\Rightarrow g=\beta\cdot\left(\sqrt{1+2/\beta}-1\right) \text{ ???}$$

However, assuming $\beta \cdot g << \beta$ and applying $\beta \cdot g \approx 0$ to (***):

$$\Rightarrow g = \sqrt{2 \cdot \beta} \Rightarrow g(n) = \sqrt{2 \cdot \beta}$$

At $g(n) = \sqrt{2 \cdot \beta} \Rightarrow E[\text{throughput}] \approx 1/\left(1 + \sqrt{2 \cdot \beta}\right)$

For $\beta \sim$ small throughput is very close to 1

However, $g(n) = \sqrt{2 \cdot \beta}$ is also small $\Rightarrow$? ***large delay???***

Pseudo-Bayesian Stabilized CSMA

Assume all packets are backlogged upon arrival and transmit with probability q_r:

$$\Rightarrow g(n) = n \cdot q_r$$

Want $g(n) = \sqrt{2 \cdot \beta}$:

$$\Rightarrow q_r(\hat{n}) = \min\left\{\frac{\sqrt{2 \cdot \beta}}{\hat{n}}, \sqrt{2 \cdot \beta}\right\} \tag{4.40}$$

$$\hat{n}_k = \begin{cases} \hat{n}_k \cdot [1 - q(\hat{n}_k)] + \lambda \cdot \beta & \text{for idle} \\ \hat{n}_k \cdot [1 - q(\hat{n}_k)] + \lambda \cdot (1 + \beta) & \text{for success} \\ \hat{n}_k + 2 + \lambda \cdot (1 + \beta) & \text{for collision} \end{cases} \tag{4.41}$$

What about the average queuing delay (W)?

$$W_i = R_i + \sum_{j=1}^{n_i} t_j + y_i \qquad (4.8), (4.42)$$

$W_i \equiv$ Total delay from the time of the i^{th} arrival until the time of the i^{th} successful transmission

$n_i \equiv$ Number of backlogged packets at time of i^{th} arrival

$R_i \equiv$ Residual slot delay at time of i^{th} arrival

$t_j \equiv$ Delay due to j^{th} departure ahead of i^{th} departure

$y_i \equiv$ Remaining interval until the beginning of the i^{th} successful transmission

E[t_j] is just the reciprocal of E[throughput], i.e., the expected number of departures per unit time:

$$E[t] = \frac{1 + \beta - e^{-g(n)}}{g(n) \cdot e^{-g(n)}} \qquad (4.44)$$

Applying Little's Theorem and taking the expectation of (4.42):

$$W = E[R] + \lambda \cdot E[t] \cdot W + E[y]$$

$$\Rightarrow W = \frac{E[R] + E[y]}{1 - \lambda \cdot E[t]}$$

E[y] is just E[t] minus the successful transmission interval:

$$E[y] = E[t] - 1 - \beta$$

$E[R]$???

Fraction of time the system spends in successful transmission intervals is $\lambda \cdot (1+\beta)$. ***Why?*** The expected residual time in these intervals is $(1+\beta)/2$.

Fraction of time spent in collisions, for small β, is negligible compared to the fraction of time spent for successful transmissions. ***Why?***

Fraction of time spent in idle period is also negligible, for small β. ***Why?***

$$\Rightarrow E[R] \approx \frac{\lambda \cdot (1+\beta)^2}{2} \qquad (4.46)$$

$$W \approx \frac{\lambda \cdot (1+\beta)^2 + 2 \cdot [E[t] - (1+\beta)]}{2 \cdot [1 - \lambda \cdot E[t]]} \qquad (4.47)$$

At $g(n) = \sqrt{2 \cdot \beta} \Rightarrow E[t] \approx 1 + \sqrt{2 \cdot \beta}$:

$$W \approx \frac{\lambda + 2 \cdot \sqrt{2 \cdot \beta}}{2 \cdot \left[1 - \lambda \cdot \left(1 + \sqrt{2 \cdot \beta}\right)\right]} \qquad (4.48)$$

For small β CSMA delay is competitive with worst-case slotted Aloha delay, $\frac{e - 1/2}{1 - \lambda \cdot e}$. ***Why???***

Q: Since $g(n) < G(n)$, how can CSMA delay be less than slotted Aloha delay???

(Hint: $g(n)$ attempted packet transmissions per *idle* slot)

CSMA Unslotted Aloha

Like slotted Aloha except that nodes are *not* synchronized to start transmissions only at the time multiples of β in idle periods

Assume a random retransmission interval that is exponential with parameter x (x, therefore, is also a *Poisson* retransmission attempt rate)

$$\Rightarrow G(n) = \lambda + n \cdot x$$

While in state n, if next packet transmission at time t is due to new arrival the system remains in state n and will transition to state $n-1$ if transmission is due to a backlogged packet

Other nodes will not detect that the channel is busy until time $t+\beta$, thus, there is the possibility of collisions:

Probability that no other node transmits = $e^{-\beta \cdot G(n)}$ or $e^{-\beta \cdot G(n-1)}$

$\Rightarrow$ Probability busy period is a collision = $1 - e^{-\beta \cdot G(n)}$ or $1 - e^{-\beta \cdot G(n-1)}$

If $\beta \cdot x$ is small, the probability of success following an idle period is approximately $e^{-\beta \cdot G(n)}$

The expected time from the beginning of one idle period to the next is:

E[time between idle periods] = $1/G(n)+1+\beta$

The $1/G(n)$ term is the expected time until the first transmission starts

The $1+\beta$ term is the time until the first transmission ends and the channel is *detected* as being idle again

If collision occurs there is a slight additional time ($<\beta$) and it is neglected here

$$\text{departure rate}(n)=\frac{e^{-\beta\cdot G(n)}}{1/G(n)+1+\beta} \qquad (4.50)$$

For small β, (4.50) is maximized when $G(n)\approx 1/\sqrt{\beta}$ yielding a maximum departure rate of approximately $1/\left(1+2\cdot\sqrt{\beta}\right)$