Chapter 4 Review Exercises (needs more problems)

1. Use separation of variables to solve

PDE	$u_t = u_{xx}$	$0 < x < 1$
BC1	$u(0,t) = 0$	$t > 0$
BC2	$u(1,t) = 0$	$t > 0$
IC	$u(x,0) = \sin 2\pi x$	$x \,\varepsilon\, [0,1]$

2. Use separation of variables to solve the problem for damped motion on a string.

PDE	$u_{xx} = \frac{1}{v^2} u_{tt} + \frac{2\pi}{L} u_t$	$0 < x < L$
BC1	$u(0,t) = 0$	$t > 0$
BC2	$u(1,t) = 0$	$t > 0$
IC1	$u(x,0) = \sin \pi x$	$x \,\varepsilon\, [0,L]$
IC2	$u'(x,0) = 0$	

3. Solve by separation of variables

PDE	$u_t = u_{xx}$	$0 < x < 1$
BC1	$u(0,t) = 0$	$t > 0$
BC2	$u(1,t) = 1$	$t > 0$
IC	$u(x,0) = x^2$	$x \,\varepsilon\, [0,1]$

4. Solve by separation of variables

PDE	$u_t = u_{xx}$	$0 < x < 1$
BC1	$u(0,t) = 0$	$t > 0$
BC2	$u(1,t) = 1$	$t > 0$
IC	$u(x,0) = \text{Sin}(\pi x) + \text{Sin}(2\pi x)$	$x \,\varepsilon\, [0,1]$

5. Use the method of Eigenfunction expansion to solve

PDE	$u_t = u_{xx} + \sin(\pi x)$	$0 < x < 1$
BC1	$u(0,t) = 0$	$t > 0$
BC2	$u(1,t) = 0$	
IC	$u(x,0) = 1$	$x \,\varepsilon\, [0,1]$

6. Solve the following problem

PDE	$u_t = u_{xx} + \sin(\pi x)$	$0 < x < 1$
BC1	$u(0,t) = 0$	$t > 0$
BC2	$u(1,t) = 1$	
IC	$u(x,0) = 1$	$x \,\varepsilon\, [0,1]$

6. Solve the following problem

PDE	$u_t = u_{xx} + \sin(\pi x)$	$0 < x < 1$
BC1	$u(0,t) = 0$	$t > 0$
BC2	$u(1,t) = 1$	
IC	$u(x,0) = \sin(\pi x)$	$x \,\varepsilon\, [0,1]$

7. Find the solution to the following problem

PDE	$u_t = u_{xx} + \delta(x - 0.5)$	$0 < x < 1$
BC1	$u(0,t) = 0$	$t > 0$
BC2	$u(1,t) = 0$	$t > 0$
IC	$u(x,0) = \sin \pi x$	$x \,\varepsilon\, [0,1]$

8. A delta function pulse of light generates charge uniformly throughout a semiconductor. Consider only positive charge for this problem. The semiconductor has volume V and the pulse produces total charge Q. Assume the equation of continuity holds $\rho_t + \nabla \cdot \vec{J} = G$ where ρ, J represent the charge density and current density, respectively.

a. Show the equation of continuity must be $\rho_t - D\rho_{xx} = \delta(t)Q/V$ for a one dimensional problem using only diffusion currents where D symbolizes a constant related to diffusion.

b. Find the solution to the following problem

PDE	$\rho_t = \rho_{xx} + \delta(t)Q/V$	$0 < x < 1$
BC1	$\rho(0,t) = 0$	$t > 0$
BC2	$\rho(1,t) = 0$	$t > 0$
IC	$\rho(x,0) = \sin \pi x$	$x \,\varepsilon\, [0,1]$

7. Suppose the previous problem includes a delta function in x. …

Need problems for cylindrical coordinates, 2-D forced drum heads, electrostatics problems.

More to be added.