330:345 Exam II Information — Fall 2004

Exam II will be based on the material from Chapters 4 and 5 covered in class and outlined in the course syllabus.

Closed book and notes. No calculators allowed.

Tables 4.1, 4.2, 5.1 and 5.2 will be distributed to the students before the exam.

Study Guide:
Do all homework problems and read the study guides given in the chapter summaries.
No proofs of the Laplace and $\mathcal{Z}$–transform properties.

Homeworks:
HW#5 Laplace Transform and Its Inverse
HW#6 Laplce Transform in System Analysis
HW#7 Z-Transform and its Inverse
HW#8 Z-Transform in System Analysis
SOLUTIONS TO ALL HOMEWORK PROBLEMS ARE POSTED ON WEBCT

Exam Time:
Thrusday, November 18, 2004; 8:10–9:30am (regular class hours)

Place: SEC 111

Attachments:
Sample Exam II
Tables 4.1, 4.2, 5.1, and 5.2

Point distribution for Exam II: (30 points = 30% of the course grade)
Chapter 4: 15 points (50%)
Chapter 5: 15 points (50%)

SPECIAL OFFICE HOURS: Wednesday, Nov. 17, 2004, 3:00–4:30pm.

332:345 — Linear Systems and Signals — Sample Exam II

#1a) 5pts. Find the Laplace transform of the following function

$$f(t) = 3t^2e^{-2t}u(t) + (t+1)e^{-t}u(t-2) + t\cos\left(\pi(t-2)\right)u(t)$$

#1b) 5pts. Find the inverse Laplace transform of the following function

$$F(s) = \frac{e^{-2s}}{s^2(s+2)^2}$$

#2a) 5pts. Find the $\mathcal{Z}$-transform of the following function

$$f[k] = 3^k(k+2)u[k-2] + \cos\left[k\frac{\pi}{2}\right]u[k-2] + f_1[k], \quad f_1[k] = \begin{cases} 5, & k=2 \\ 9, & k=4 \\ 0, & \text{otherwise} \end{cases}$$

#2b) 5pts. Find the inverse $\mathcal{Z}$-transform of the following function

$$F(z) = \frac{3z(z+1)}{(z+2)(z+4)(z+6)}$$

Using the initial and final values theorems find $f[0]$ and $f[+\infty]$.

#3a) 5pts. Consider a continuous-time system

$$\frac{d^2y(t)}{dt^2} + 2\frac{dy(t)}{dt} + y(t) = \frac{df(t)}{dt} + 3f(t)$$
$$y\left(0^-\right) = 1, \quad y^{(1)}\left(0^-\right) = 2, \quad f(t) = tu(t)$$

Find its transfer function (1pt), impulse response (1pt), step response (1pt), zero-state (1pt), and zero-input (1pt) responses.

#3b) 5pts. Consider a discrete-time system

$$y[k+2] + \frac{1}{6}y[k+1] - \frac{1}{6}y[k] = f[k+1] + 2f[k], \qquad y[0] = 1, \ \ y[1] = 2$$

with $f[k] = (-1)^k u[k]$. Find its transfer function (1pt), impulse response (1pt), step response (1pt), and complete response (2pts).

Hint: $\cos(\alpha \pm \beta) = \cos(\alpha)\cos(\beta) \mp \sin(\alpha)\sin(\beta)$.

Table 4.1: Properties of the Laplace Transform

$\mathcal{L}\{\alpha_1 f_1(t) + \alpha_2 f_2(t)\}$	$\alpha_1 F_1(s) + \alpha_2 F_2(s)$
$\mathcal{L}\{f(t-t_0)u(t-t_0)\}$	$e^{-st_0}F(s), \quad t_0 > 0$
$\mathcal{L}\{f(at)\}$	$\frac{1}{a}F\left(\frac{s}{a}\right), \quad a > 0$
$\mathcal{L}\{t^n f(t)\}$	$(-1)^n \frac{d^n}{ds^n}F(s)$
$e^{\lambda t}f(t)$	$F(s-\lambda)$
$f(t)\cos(\omega_0 t)$ $f(t)\sin(\omega_0 t)$	$\frac{1}{2}[F(s+j\omega_0) + F(s-j\omega_0)]$ $\frac{j}{2}[F(s+j\omega_0) - F(s-j\omega_0)]$
$\mathcal{L}\left\{\frac{d}{dt}f(t)\right\}$	$sF(s) - f\left(0^-\right)$
$\mathcal{L}\left\{\frac{d^2}{dt^2}f(t)\right\}$	$s^2F(s) - sf\left(0^-\right) - f^{(1)}\left(0^-\right)$
$\mathcal{L}\left\{\frac{d^n}{dt^n}f(t)\right\}$	$s^nF(s) - s^{n-1}f\left(0^-\right) - s^{n-2}f^{(1)}\left(0^-\right)$ $- \cdots - f^{(n-1)}\left(0^-\right)$
$\mathcal{L}\{f_1(t) * f_2(t)\}$	$F_1(s)F_2(s)$
$\mathcal{L}\{\int_0^t f(\tau)d\tau\}$	$\frac{1}{s}F(s)$
$\lim_{t\to 0^+}\{f(t)\}$	$\lim_{s\to\infty}\{sF(s)\}$
$\lim_{t\to\infty}\{f(t)\}$	$\lim_{s\to 0}\{sF(s)\}$

Table 4.2: Common Laplace transform pairs

$\delta(t)$	1
$u(t)$	$\frac{1}{s}$
$e^{-\alpha t}u(t)$	$\frac{1}{s+\alpha}$
$t^n u(t)$	$\frac{n!}{s^{n+1}}$
$t^n e^{-\alpha t}u(t)$	$\frac{n!}{(s+\alpha)^{n+1}}$
$u(t)\cos(\omega t)$	$\frac{s}{s^2+\omega^2}$
$u(t)\sin(\omega t)$	$\frac{\omega}{s^2+\omega^2}$
$e^{-\alpha t}u(t)\cos(\omega t)$	$\frac{s+\alpha}{(s+\alpha)^2+\omega^2}$
$e^{-\alpha t}u(t)\sin(\omega t)$	$\frac{\omega}{(s+\alpha)^2+\omega^2}$
$tu(t)\cos(\omega t)$	$\frac{s^2-\omega^2}{(s^2+\omega^2)^2}$
$tu(t)\sin(\omega t)$	$\frac{2\omega s}{(s^2+\omega^2)^2}$
$te^{-\alpha t}u(t)\cos(\omega t)$	$\frac{(s+\alpha)^2-\omega^2}{\left((s+\alpha)^2+\omega^2\right)^2}$
$te^{-\alpha t}u(t)\sin(\omega t)$	$\frac{2\omega(s+\alpha)}{\left((s+\alpha)^2+\omega^2\right)^2}$

Table 5.1: Properties of the $\mathcal{Z}$-transform

$\mathcal{Z}\{a_1 f_1[k] \pm a_2 f_2[k]\}$	$a_1 F_1(z) \pm a_2 F_2(z)$
$\mathcal{Z}\{f[k-k_0]u[k-k_0]\}$	$\frac{1}{z^{k_0}} F(z)$
$\mathcal{Z}\{f[k-1]u[k]\}$	$\frac{1}{z} F(z) + f[-1]$
$\mathcal{Z}\{f[k-2]u[k]\}$	$\frac{1}{z^2} F(z) + \frac{1}{z} f[-1] + f[-2]$
$\mathcal{Z}\{f[k-k_0]u[k]\}$	$\frac{1}{z^{k_0}} F(z) + \frac{1}{z^{k_0-1}} f[-1] + \cdots + \frac{1}{z} f[-k_0+1] + f[-k_0]$
$\mathcal{Z}\{f[k+1]u[k]\}$	$zF(z) - zf[0]$
$\mathcal{Z}\{f[k+2]u[k]\}$	$z^2 F(z) - z^2 f[0] - zf[1]$
$\mathcal{Z}\{f[k+k_0]u[k]\}$	$z^{k_0} F(z) - z^{k_0} f[0] - z^{k_0-1} f[1] - \cdots - zf[k_0-1]$
$\mathcal{Z}\{kf[k]\}$	$-z \frac{d}{dz} F(z)$
$\mathcal{Z}\left\{k^2 f[k]\right\}$	$z \frac{d}{dz} F(z) + z^2 \frac{d^2}{dz^2} F(z)$
$\mathcal{Z}\left\{a^k f[k]\right\}$	$F\left(\frac{z}{a}\right)$
$\mathcal{Z}\{f[k]\cos(\omega kT)\}$	$\frac{1}{2}\left[F\left(ze^{j\omega T}\right) + F\left(ze^{-j\omega T}\right)\right]$
$\mathcal{Z}\{f[k]\sin(\omega kT)\}$	$\frac{j}{2}\left[F\left(ze^{j\omega T}\right) - F\left(ze^{-j\omega T}\right)\right]$
$\mathcal{Z}\{f_1[k] * f_2[k]\}$	$F_1(z) F_2(z)$
$\lim\limits_{k \to 0} f[k]$	$\lim\limits_{z \to \infty} \{F(z)\}$
$\lim\limits_{k \to \infty} f[k]$	$\lim\limits_{z \to 1} \left\{\frac{z-1}{z} F(z)\right\}$

Table 5.2: Common $\mathcal{Z}$-transform pairs

$\delta[k]$	1
$u[k]$	$\frac{z}{z-1}$
$a^k u[k]$	$\frac{z}{z-a}$
$ku[k]$	$\frac{z}{(z-1)^2}$
$k^2 u[k]$	$\frac{z(z+1)}{(z-1)^3}$
$ka^k u[k]$	$\frac{az}{(z-a)^2}$
$k^2 a^k u[k]$	$\frac{az(z+1)}{(z-a)^3}$
$\frac{1}{m!}k(k-1)(k-2)\cdots(k-m+1)u[k]$	$\frac{z}{(z-1)^m}$
$\frac{1}{m!}k(k+1)(k+2)\cdots(k+m)a^k u[k]$	$\frac{z^{m+1}}{(z-a)^{m+1}}$
$u[k]\cos(\omega kT)$	$\frac{z^2-z\cos(\omega T)}{z^2-2z\cos(\omega T)+1}$
$u[k]\sin(\omega kT)$	$\frac{z\sin(\omega T)}{z^2-2z\cos(\omega T)+1}$
$a^k u[k]\cos(\omega kT)$	$\frac{z^2-az\cos(\omega T)}{z^2-2az\cos(\omega T)+a^2}$
$a^k u[k]\sin(\omega kT)$	$\frac{az\sin(\omega T)}{z^2-2az\cos(\omega T)+a^2}$