3.5 State Space MATLAB Laboratory Experiments

In this section we present three MATLAB laboratory experiments on the state space method in control systems. These experiments can be used either as supplements for lectures or independently in the corresponding control system laboratory. Most of the required MATLAB functions have been already introduced in the examples done in this chapter. Students should also consult Appendix D, where a shortened MATLAB manual is given. It is advisable that before using any MATLAB function, the students check all its options by typing `help function name`.

3.5.1 Experiment 1—The Inverted Pendulum

Part 1. The linearized equations of the inverted pendulum, obtained by assuming that the pendulum mass is concentrated at its center of gravity (Kwakernaak and Sivan, 1972; Kamen, 1990) are given by

$$\begin{aligned} \left(J + mL^2\right)\ddot{\theta}(t) - mgL\theta(t) + mL\ddot{d}(t) &= 0 \\ (M + m)\ddot{d}(t) + mL\ddot{\theta}(t) &= u(t) \end{aligned} \tag{3.107}$$

where $\theta(t)$ is the angle of the pendulum from the vertical position, $d(t)$ is the position of the cart, $u(t)$ is the force applied to the cart, M is the mass of the cart, m is the mass of the pendulum, g is the gravitational constant, and J is the moment of inertia about the center of mass. Assuming that normalized values are given by $J = 1$, $L = 1$, $g = 9.81$, $M = 1$, $m = 0.1$, derive the state space form

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}u(t)$$

where

$$\mathbf{x}(t) = [\theta(t) \quad \dot{\theta}(t) \quad d(t) \quad \dot{d}(t)]^T$$

and $\mathbf{A}^{4\times4}$ and $\mathbf{B}^{4\times1}$ are the corresponding matrices.

Part 2. Using MATLAB determine the following:

(a) The eigenvalues, eigenvectors, and characteristic polynomial of matrix $\mathbf{A}$.

(b) The state transition matrix at the time instant $t = 1$.

(c) The unit impulse response (take $\theta(t)$ and $d(t)$ as the output variables) for $0 \leq t \leq 1$ with the step size $\Delta t = 0.1$ and draw the system response using the MATLAB function `plot`.

(d) The unit step response for $0 \leq t \leq 1$ and $\Delta t = 0.1$. Draw the system response.

(e) The unit ramp response for $0 \leq t \leq 1$ and $\Delta t = 0.1$ and draw the system response. Compare the response diagrams obtained in (c), (d), and (e).

(f) The state response resulting from the initial state $\mathbf{x}(0) = [-1 \quad 1 \quad 1 \quad 1]^T$ and the input $\mathbf{u}(t) = \sin(t)$ for $0 \leq t \leq 5$ and $\Delta t = 0.1$.

(g) The inverse of the state transition matrix $\left(e^{\mathbf{A}t}\right)^{-1}$ for $t = 5$.

(h) The state $\mathbf{x}(t)$ at time $t = 5$ assuming that $\mathbf{x}(10) = [10 \quad 0 \quad 5 \quad 2]^T$ and $u(t) = 0$ by using the result from (g).

(i) Find the system transfer function.

Part 3. Discretize the continuous-time system given in (3.107) with $T = 0.02$ and find the discrete space model

$$\mathbf{x}(k+1) = \mathbf{A}_d\mathbf{x}(k) + \mathbf{B}_d u(k)$$

Assuming that the output equation of the discrete system is given by

$$\mathbf{y}(k) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \mathbf{x}(k) = \mathbf{C}_d\mathbf{x}(k)$$

find the system (output) response for $0 \leq k \leq 50$ due to initial conditions $\mathbf{x}_0 = [-1 \quad 1 \quad -1 \quad 1]^T$ and unit step input (note that $u(k)$ should be generated as a column vector of 50 elements equal to 1).

Part 4. Consider the continuous-time system given by

$$\frac{d^2y(t)}{dt^2} + 0.1\frac{dy(t)}{dt} = u(t) \tag{3.108}$$

(a) Discretize this system with $T = 1$ by using the Euler approximation.

(b) Find the response of the obtained discrete system for $k = 1, 2, 3, ..., 20$, when $u(t) = sin(0.1\pi t)$ and $y(0) = \dot{y}(0) = 0$.

(c) Find discrete transfer function, characteristic equation, eigenvalues, and eigenvectors.

Part 5. Discretize the state space form of (3.108) obtained by using MATLAB function `c2d` with $T = 1$. Find the discrete system response for the initial condition and the input function defined in Part 4b. Compare the results obtained in Parts 4 and 5. Comment on the results obtained.