PID Controllers in Nineties

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Overview

◆ Purpose: extract the essence of the most recent development of PID control
◆ Based on the survey of papers (333) in nineties in the following journals:
  - IEEE Transactions on Automatic Control (23)
  - IEEE Transactions of Control Systems Technology (26)
  - IEEE Transactions on Robotic and Automation (11)
  - IEEE Transactions on Industrial Electronics (2)
  - IEEE Control Systems Magazine (4)
  - IFAC Automatica (59)
  - IFAC Control Engineering Practice (29)
  - International Journal of Control (20)
  - International Journal of Systems Science (2)
  - International Journal of Adaptive Control and Signal Processing (2)
  - IEE Proceedings - Control Theory and Applications (30)
Overview

Based on the survey of papers (333) in nineties:

- Journal of the Franklin Institute (5)
- Control and Computers (1)
- Computing & Control Engineering Journal (3)
- Computers and Chemical Engineering (1)
- AIChE Journal (16)
- Chemical Engineering Progress (2)
- Chemical Engineering Communications (1)
- Industrial and Engineering Chemistry Research (55)
- ISA Transactions (21)
- Journal of Process Control (13)
- Transactions of ASME (1)
- ASME Journal of Dynamic Systems, Measurements and Control (3)
- Electronics Letters (2)
- Systems & Control Letters (1)
Paper Classification

- Ziegler-Nichols based PIDs (10)
- Frequency domain based PIDs (22)
- Relay based PIDs (29)
- Optimization methods based PIDs (20)
- Internal Model Control PIDs (15)
- Robust PID controllers (30)
- Nonlinear PIDs (12)
- Adaptive PIDs (28)
- Anti-windup techniques (13)
- Neural Network/Fuzzy Logic based PIDs (34)
- PID control of Distributed Systems (3)
- Multivariable PIDs (29)
- Applications of PID controllers (56)
Forms of PID controller

<table>
<thead>
<tr>
<th>Form</th>
<th>Equation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Standard form</strong></td>
<td>[ u = K_p \left( e + \frac{1}{T_i} \int e dt + T_d \frac{de}{dt} \right) ]</td>
<td>( e = y_r - y )</td>
</tr>
<tr>
<td>( G_s(s) = K_p \left( 1 + \frac{1}{T_i s} + T_d s \right) )</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Cascade form</strong></td>
<td>[ G_c(s) = K \left( 1 + \frac{1}{T_i s} \right) \left( 1 + T_d s \right) ]</td>
<td>( G_c(s) = k + \frac{k_i}{s} + k_d s )</td>
</tr>
<tr>
<td><strong>Parallel form</strong></td>
<td>[ G_c(s) = K \left( 1 + \frac{1}{T_i s} \right) \left( 1 + T_d s \right) ]</td>
<td>( G_c(s) = k + \frac{k_i}{s} + k_d s )</td>
</tr>
<tr>
<td><strong>Filtered derivative term</strong></td>
<td>[ u_c = k_c \left( e + \frac{1}{T_i} \int e dt - T_d \frac{dy_f}{dt} \right) ]</td>
<td>( e = y_r - y )</td>
</tr>
<tr>
<td>( y_f = \frac{1}{1 + s T_d / N} y )</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Weighted setpoint form</strong></td>
<td>[ u_c = k_c \left[ (\beta y_r - y) + \frac{1}{T_i} \int e dt - T_d \frac{dy_f}{dt} \right] ]</td>
<td></td>
</tr>
</tbody>
</table>
Modeling (step response)

\[ G(s) = \frac{a}{s\tau} e^{-\tau s} \quad G(s) = \frac{K_p e^{-s\tau}}{1 + sT} \]

\[ T + \tau = \frac{A_0}{K_p} \]

\[ T = \frac{A_1}{K_p} e^1 \]
Modeling (frequency domain)

\[ N(a) = \frac{4d}{\pi a} \]

(a) Relay Excitation

(b) Correlation Method:
- use PRBS test signal \( u(t) \),
- measure \( y(t) \),
- find cross-correlation function between \( u(t) \) and \( y(t) \)
- compute the impulse response \( g(t) \)
- transform \( g(t) \) to \( G(s) \) and find the parameters of the model
Tuning Techniques

- Ziegler-Nichols (10)
- Frequency domain tuning (22)
- Relay based tuning (29)
- Tuning using optimization (20)
- Internal model control tuning (15)
- Other tuning techniques (30)
Ziegler-Nichols Tuning

- Originated by work of Ziegler and Nichols, 1942
- Still in broad industrial use
- Several improvements reported
- Controllers tuned by this method tend to have large overshoot
- Two methods - time and frequency domain based

\[
u_c = k_c \left[ (\beta y_r - y) + \frac{1}{T_i} \int edt - T_d \frac{dy_f}{dt} \right]
\]

\[0 < \beta < 1\]

<table>
<thead>
<tr>
<th>Controller</th>
<th>K</th>
<th>T_i</th>
<th>T_d</th>
</tr>
</thead>
<tbody>
<tr>
<td>P</td>
<td>1/a</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PI</td>
<td>0.9/a</td>
<td>3L</td>
<td></td>
</tr>
<tr>
<td>PID</td>
<td>1.2/a</td>
<td>2L</td>
<td>L/2</td>
</tr>
</tbody>
</table>
Ziegler-Nichols Freq. Response

\[ \frac{1}{K} \left( 1 + j \frac{\omega}{\omega_c} \right) \]

**PID** | **PI**
---|---
Proportional gain | \( k_c = 0.6k_u \) \( k_c = 0.45k_u \)
Integral time | \( T_i = 0.5t_u \) \( T_i = 0.85t_u \)
Derivative time | \( T_d = 0.125t_u \)
**Refined Ziegler-Nichols**

\[ \kappa = k_p k_u \]

Based on normalized parameters:
\[ \Theta = \frac{\theta_a}{T_p} = \frac{a}{k_p} \]

<table>
<thead>
<tr>
<th>Refined Ziegler-Nichols formulae for PID control</th>
<th>PID</th>
</tr>
</thead>
<tbody>
<tr>
<td>Large normalized Process gain or small normalized dead time (2.25 &lt; \kappa &lt; 15; 0.16 &lt; \Theta &lt; 0.57)</td>
<td>(k_c = 0.6k_u)</td>
</tr>
<tr>
<td>(\beta = \frac{15 - \kappa}{15 + \kappa}) (10% overshoot)</td>
<td>(T_i = 0.5\mu t_u)</td>
</tr>
<tr>
<td>(\beta = \frac{36}{27 + 5\kappa}) (20% overshoot)</td>
<td>(T_d = 0.125t_u)</td>
</tr>
<tr>
<td>Small normalized process gain or large normalized dead time (1.5 &lt; \kappa &lt; 2.25; 0.57 &lt; \Theta &lt; 0.96)</td>
<td>(\mu = \frac{4}{9} \kappa; \ \beta = \frac{8}{17\left(\frac{4}{9} \kappa + 1\right)}) (20% overshoot and 10% undershoot)</td>
</tr>
</tbody>
</table>
Frequency Domain Tuning Techniques

- Variety of the techniques based on desired phase and gain margin, and other frequency response parameters:
  - Hagglund & Astrom 1992;
  - Tyreus & Luyben, 1992;
  - Venkatashankar & Chidambaram, 1994;
  - Luyben, 1996, 1998;
  - Khan & Leman, 1996;
  - Poulin & Pomerlau, 1996;
  - Loron, 1997;
  - Shafei & Shenton, 1997;
  - Natarjan & Gilbert, 1997;
Relay Based Tuning Techniques

- Introduced by (Astrom and Hagglund, 1994)
- Considered in many papers
- Relay Tuning considering two-parameter nonlinearity (Friman and Waller, 1995)
- Enhanced relay tuning by using the estimate at the two points of the Nyquist plot (Sung and Lee, 1997)
- Relay tuning that identifies three frequency data sets (Tan et al., 1996) using one feedback relay test
- Multiple-point frequency response fitting based on relay tuning (Wang et al., 1999)
- Two relays working in parallel (Friman and Waller, 1997)
- A specialized book on relay tuning (Yu, 1999)
Relay tuning (Astrom, Hagglund) is one of the most important methods commercially used.

\[ 1 + N(a, \omega)G(j\omega) = 0 \quad \Rightarrow \quad G(j\omega) = -\frac{1}{N(a, \omega)} \]

\[ N(a) = \frac{4d}{\pi a} \]

\[ \text{Re}\{G(j\omega)\} = -\frac{1}{N(a)} \quad , \quad \text{Im}\{G(j\omega)\} = 0 \]

\[ -\frac{1}{K_c} = -\frac{1}{N(a)} = -\frac{\pi a}{4d} \quad \Rightarrow \quad K_c = \frac{4d}{\pi a} \]
Types of Relays

Ideal Relay

 Relay with Hysteresis

 Saturation Relay

\[ N(a) = \frac{4d}{\pi a} \]

\[ N(a) = \frac{4d}{\pi a} \left[ \sqrt{a^2 - \varepsilon^2} - j\varepsilon \right] \]

\[ N(a) = \frac{2d}{\pi} \left( \frac{1}{\bar{a}} \sin^{-1} \frac{\bar{a}}{a} + \sqrt{\frac{a^2 - \bar{a}^2}{a^2}} \right) \]
Limit Cycle Parameters

Ideal/Saturation Relay

\[ \frac{1}{K} \text{Re}\{G(j\omega)\} \]

\[ j \text{Im}\{G(j\omega)\} \]

\[ (1/K, j0) \]

\[ \omega = \omega_c \]

Relay with hysteresis

\[ \phi_m = \tan^{-1}\left( \frac{\varepsilon}{\sqrt{a^2 - \varepsilon^2}} \right) \]

\[ (1/K, j0) \]

\[ \omega = \omega_c \]
Ideal and Saturation Relay

\[ G(s) = \frac{12.8e^{-s}}{16.8s + 1} \]
System with RHP Zeros

\[ G(s) = \frac{(-3s + 1)e^{-0.6s}}{(5s + 1)(s + 1)} \]
System with two RHP zeros

$$G(s) = \frac{(-s + 1)^2 e^{-0.1s}}{(0.8s + 1)^3}$$

Wrong sign of the system

Switched at b

Switched at c
Load Disturbance Effect

\[ G(s) = \frac{e^{-1.5s}}{(10s + 1)(s + 1)} \]

\[ G_L(s) = \frac{e^{-s}}{5s + 1} \]
Multiple point estimation

- Time delay element in series with a relay

(Besancon-Voda and Roux, Buisson, 1997)

(Schei, 1992)
Tuning Using Optimization Methods

- Based on optimization of certain, mostly integral criteria
- The technique dates back to papers (Johnson, 1968; Athans, 1971; Williamson & Moore, 1971)

\[
J_n(\theta) = \int_0^\infty [t^n e(\theta, t)]^2 dt
\]

- Most of the methods based on FOPD system
- PID tuned in frequency domain using an optimization (Liu and Dailey, 1999)
- Comparative study (Ho et al., 1999)
Internal Model Control Tuning

- Developed by Morari and co-workers (Garcia and Morari, 1982)
- IMC is a general design technique - PID is a special case
- This is analytical method of PID design based on FOPD model.
- Tuning by this method considered in (Chien & Fruehauf, 1990; Rotstein & Levin, 1991; Jacob & Chidambaram, 1996).
- Comparative study between IMC based and frequency based tuning (Hang et al., 1994)
- Several IMC schemes compared in (Vandeursen & Peperstraete, 1996)
- IMC has very good robustness (Scali et al., 1992)
- Simplified tuning rules for IMC presented in (Fruehauf et al., 1994)
- Improved filter design for IMC proposed in (Horn, 1996).
Other Tuning Methods

- Approximation of pure time delay by Pade approximation (Yutawa & Seborg, 1982) of FOPD model to get second order system.
- Iterative technique to solve transcendental equation (Lee, 1989)
- Pattern recognition based adaptive controller (Cao & McAvoy, 1990)
- Transient response of second order plus time delay (Hwang, 1995)
- Graphical tuning based on the parametric D-stability partitioning (Shafei & Shenton, 1994)
- Gain scheduling tuning (McMillan et al., 1994)
- Tuning based on the closed-loop system specification (Abbas, 1994)
- Delay compensation PID tuning formula based on Smith predictor (Tsang et al., 1994)
- Pole-placement method (Hwang & Shiu, 1994)
- Model-based PID tuning (Huang et al., 1996)
- Kessler's Symmetric optimum principle (Voda & Landau, 1995)
Kessler’s Symmetrical Optimum Principle

- Based on two Kessler’s papers from fifties which describe PID design technique based on Bode diagrams.
- The idea is based on the idea that the plant transfer function be as close as possible to one at low frequency by accommodating and at as high as possible.
- Kessler’s principle says that:
  - The gain cross over frequency of the compensated system should be placed at \( \omega_{cg} = 1/2\tau_e \), where \( \tau_e \) is equivalent time constant of all noncompensable time constants (sum of fast time constants and time delay).
  - The slope of the Bode diagram at the gain crossover frequency is minus 20 dB/dec.
  - The PID controller is chosen such that it preserves the slope of minus 20 dB/dec for one octave to the right and \( m \) octaves to the left (\( m \) is the number of compensated time constants).
**Kessler’s Symmetrical Optimum Principle**

\[
G(s) = \frac{Ke^{-s\tau_n}}{(1+\tau_1s)(1+\tau_2s)...(1+\tau_{n-1}s)(1+\tau_n)} \approx G_{app}(s) = \frac{K}{(1+\tau_1s)(1+\tau_2s)(1+\tau_es)}, \quad \omega \leq \frac{1}{\tau_e}
\]

For \( m=2 \) and \( \tau_1 \geq \tau_2 >> (\tau_3 + \tau_4 + ... + \tau_n) = \tau_e \)

In the neighborhood of the gain crossover frequency, \( \omega_{cg} = 1/2\tau_e \), \( G(s) \) is approximated by

\[
G(s) = \frac{K}{(1+\tau_1s)(1+\tau_es)}, \text{ with } \tau_1 \geq 4\tau_e
\]

Tuning of PID controller by Kessler’s method (Voda and Landau, 1995)

<table>
<thead>
<tr>
<th>Controller Type</th>
<th>Assumed Model</th>
<th>Controller Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>PI</td>
<td>( G_1(s) = \frac{K}{(1+\tau_1s)(1+\tau_es)}, \tau_1 \geq \tau_e )</td>
<td>( K_p = \frac{0.5\tau_1}{K\tau_e}, \quad T_i = 4\tau_e )</td>
</tr>
<tr>
<td>PID</td>
<td>( G_2(s) = \frac{K}{(1+\tau_1s)(1+\tau_2s)(1+\tau_es)}, \tau_1, \tau_2 \geq \tau_e )</td>
<td>( T_d = \frac{4\tau_2\tau_e}{\tau_2 + 4\tau_e}, \quad T_i = \tau_2 + 4\tau_e, \quad K_p = \frac{\tau_1(\tau_2 + 4\tau_e)}{8K\tau_e^2} )</td>
</tr>
</tbody>
</table>
Kessler’s method salient features

- Produces good phase and gain margins by imposing the slope 20 dB/dec around the gain crossover frequency
- Handles well nonlinearities and time varying parameters, and takes into account unmodeled dynamics (represented by the equivalent time constant $\tau_e$ (Voda and Landau, 1995)
- The frequency $1/\tau_e$ can be found from the Nyquist diagram where the phase margin is around 45°. This frequency also represents the closed loop bandwidth
- This frequency can be determined from a relay with hysteresis feedback experiment, as follows:

**PID Tuning by Kessler-Landau-Voda method (KLV)**

<table>
<thead>
<tr>
<th>Controller Type</th>
<th>Assumption</th>
<th>Controller Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>PI</td>
<td>$\omega_{135} = \frac{\alpha}{\tau_e}$</td>
<td>$K_p = \frac{1}{3.5\tau_e}$, $T_i = \frac{4\alpha}{\omega_{135}} = \frac{4.6}{\omega_{135}}$</td>
</tr>
<tr>
<td>PID</td>
<td>$\tau_2 \approx 1/\omega_{135}$ $\Rightarrow \omega_{135} = 1/\beta\tau_e$, $1 &lt; \beta &lt; 2$</td>
<td>$K_p = \frac{\beta(4 + \beta)}{8\sqrt{2}G(\omega_{135})}$, $T_i = \frac{4 + \beta}{\beta\omega_{135}}$, $T_d = \frac{4}{(4 + \beta)\omega_{135}}$</td>
</tr>
</tbody>
</table>
Industrial Controllers

- ABB Commander 355:
  - Gain scheduling, feedforward, cascade, ratio control, autotune for 1/2 wave od minimal overshoot

- Foxboro 762C:
  - Exact Self-tuning control, dynamic compensation: lead/lag, impulse, dead time.

- Fuji Electric PYX:
  - Autotuning, fuzzy logic feedback control

- Honeywell (few different models):
  - Self-tuning, autotuning, gain scheduling, fuzzy logic overshoot suppression

- Yokogawa:
  - Autotuning, overshoot suppression (at sudden change of setpoint), gain scheduling
Implementation Issues

- Commercial controllers
  - Of the shelf units
  - Mostly digital versions with sophisticated auto-tuning features
  - Used in SISO (or multi-loop control architectures)
  - Give satisfactory results (according to the Corning engineers)
  - Digital controllers have 0.1s sampling period - good for process control
  - Contain many of additional features based on many years of application experience (integral windup prevention, integral preload, derivative limiting, bumpless transfer)
  - The above features make PID safe to use.
Embedded Controllers

- Customized to the specific needs (when there are special requirements for speed, size, ...)
- Needed when the custom version of PID control (combined with monitoring, alarm processing, communication software ... if needed)
- Can accommodate virtually any tuning method
- Very fast control loops require fixed-point arithmetic and special electronics for implementation (DSP, FPGA,...)
- Example: optical amplifier gain and output optical power control - needs very fast sampling rates.
Conclusions

◆ PID (PD, PI) controllers received lot of attention during ‘90s.
◆ Centennial work of Ziegler and Nichols (1942) still widely used in industrial applications and as a benchmark for new techniques.
◆ Despite of a huge number of theoretical and application papers on tuning techniques of PID controllers, this area still remains open for further research.
◆ There is lack of comparative analysis between different tuning techniques.
◆ No common benchmark examples.
◆ There is a number of industrial controllers based on modern tuning techniques.
◆ Embedded controllers are good candidates for new PID techniques.
◆ The area is still open for research.