

Section 6.1: A Brief Overview of the Quantum Theory of Electromagnetic Fields

As well known, atoms can emit light waves that are coherent with a driving optical field (stimulated emission) and they can also emit light on their own without a driving field (spontaneous emission). The spontaneous emission arises as a result of quantum vacuum fluctuations. We need to introduce quantum electrodynamics in order to discuss vacuum fluctuations.

Maxwell's electromagnetic (EM) equations can be solved to define a set of allowed EM modes. Sine and cosine functions represent the allowed modes for a cubic volume; for example, imagine sinusoidal standing waves in a Fabry-Perot cavity. The allowed wavelengths and polarization of light characterize these modes. The fields additionally have amplitude and phase; these attributes characterize only the field and not the mode. In QED, the electric field becomes an operator having the *form*

$$\hat{\mathcal{E}} \sim \hat{q} \text{Sin}(kz - \omega t) + \hat{p} \text{Cos}(kz - \omega t)$$

for a single mode. This can be recognized as an alternative to writing the field in terms of the amplitude and phase. The quadrature operators \hat{q}, \hat{p} refer to amplitude and not to position and momentum although people often refer to them in such a manner. The operators do not commute and must operate on vectors in a Hilbert space that describe the amplitude and phase of the field. The various vectors in the amplitude space lead to the various EM fields with distinct properties. The states of light refer to the basis states of the amplitude space or to various combinations of the basis states. The Fock, coherent and squeezed states represent three types of amplitude states.

The QED *Fock* state represents one of the most fundamental notions of Quantum ElectroDynamics (QED). A Fock state has a definite number of photons in the mode (this means that each mode has a definite average power) but completely random phase. In some sense, the photons occupy the modes, which function as a type of framework or stage. In *classical* electrodynamics, a state without any photons corresponds to a mode without any amplitude. In QED, a state without any photons (the vacuum state) has an *average* electric field of zero, but non-zero variance (which is proportional to the square of the field). This means that the value of the electric field can fluctuate away from the average of zero. The non-zero variance refers to quantum fluctuations or noise; the vacuum state has the minimum quantum noise often termed vacuum fluctuations. Fock states make it easy to count photons but there exists a slight complication for engineering purposes! It turns out that *all* Fock states have zero average electric field because of the random phase. In addition, the noise associated with the Fock state must be larger than the minimum value set by the vacuum.

A *coherent* state has nonzero average electric field and fairly well defined phase. The electric fields for these states can be pictured as sine and cosine waves; these states best describe laser emission. The coherent state actually consists of a linear combination of all Fock states. Coherent and Fock states can be seen to be quite different. One of the most important distinctions is that, for a coherent state with given amplitude, a Poisson probability distribution describes the number of photons n in the mode. A Fock state has an exact number of photons. The Poisson probability distribution links the standard deviation of photon number $\sigma = \sqrt{\langle n \rangle}$ with the average number of photons $\langle n \rangle$. For example, a beam with an average of $\langle n \rangle = 100$ photons has a standard deviation of $\sqrt{\langle n \rangle} = 10$ photons. One might reasonably expect the measured number of photons to

range from 80 to 120 (almost 50% variation). The variation represents the shot noise. Now returning to the amplitude and phase, it just so happens that any coherent state has the same noise content as the vacuum state (regardless of the amplitude of the coherent state).

A *squeezed* vacuum state can be produced from the quantum vacuum state by reducing the noise (i.e., reducing the variance) in one set of parameters while adding it to another (i.e., “squeezing the noise out”). Squeezing the vacuum state is equivalent to squeezing the coherent state since the vacuum and coherent states have the same type and amount of noise. For example, noise can be removed from one quadrature term of the electric field for “quadrature squeezing” but that removed noise reappears in the other quadrature term. Similarly, a “quiet” photon stream (i.e. a number squeezed state) obtains by removing noise from the photon-number but it reappears in the phase. “Sub-Poisson” statistics describe the quiet photon stream. Phase-squeezed states have less phase noise but more amplitude noise. Squeezed coherent states can be produced, detected and used for low noise applications. Figure 4.1.1 shows examples of laser light moving past an observer. The top portion shows “coherent light” (i.e., light in a coherent state) where the number of photons in equal beam-lengths can vary from one length-interval to the next. The number of photons follows the Poisson probability distribution. The bottom portion of the figure shows a “number squeezed state” where the equal lengths have equal numbers of photons. Apparently, a number-squeezed state is related to a Fock state.

Spontaneous emission comprises another form of noise in the laser although we certainly should not term it as “noise” for a Light Emitting Diode (LED). We require spontaneous emission in a laser to start the laser oscillation but, in addition to producing larger than necessary threshold current, it also wastes energy. Interestingly, spontaneous emission is not solely a property of a collection of atoms, but arises from quantum vacuum fluctuations. The fluctuating electric field of the vacuum state initiates the spontaneous emission. Changing the number of vacuum modes coupled to the atomic ensemble can modify the rate of spontaneous emission – there exists one vacuum mode for each wavelength and polarization allowed by the boundary conditions on the enclosed volume. The field of Cavity QED describes the theory and measurement of both spontaneous and stimulated emission for which these interesting cavity effects become important. These vacuum effects are essential for emitters (LEDs or Lasers) that have physical sizes comparable to the wavelength of the emitted light (nanophotonics). Further, to characterize the effect of spontaneous emission on another laser or device, it is necessary to understand the effects of vacuum entropy.

Noise can be a problem because small (and low power) components do not deal with many particles (electrons, holes and photons) at one time. For low particle numbers, as might be typical for small or low power components, the uncertainty (or standard deviation) in the signal can be roughly the same size as the magnitude of the quantity itself. Equivalently stated, small systems and signals have relatively large deviation of

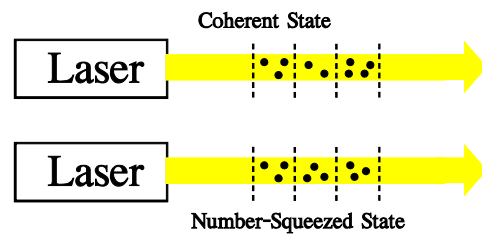


Figure 4.1.1: Artist view of the coherent and the number-squeezed states.

the number of particles carrying the signal compared with the average number. Ultimately, nanometer-scale devices (and for low power systems) have many different types of noise with the quantum noise representing the commonly accepted lowest noise floor.

Noise can be more detrimental to an analog signal than a digital one. An analog signal usually carries information of a continuously varying parameter (such as distance, length, temperature or music) and therefore, the noise determines the ultimate precision of the measurement or the quality of the impressed information. Noise as small as 0.1% can be significant for audio applications (for example). A digital system, however, must be capable of distinguishing between a logic “0” and “1”. The signal strength must exceed a threshold value before the circuit recognizes the logic level. Many circuits and devices include a hysteresis effect to reduce the affect of noise. The “bit error rate” determines the accuracy of the digital system.

Noise problems can also appear in low power, high frequency RF or RADAR transmitters. These transmitters must operate at higher powers in order to keep the signal-to-noise ratios S/N as large as possible. For conventional electronic equipment (not just optical equipment) operating at modest powers of 10 W and 30 GHz, the quantum noise becomes a significant factor over a distance of 5 miles.

The theory of quantized fields mathematically unifies the pictures of light as particles and as waves. We know the photon as the basic quantum of light. The EM fields and the EM Hamiltonian are quantized similar to the electronic harmonic oscillator. The quantized electric field will be seen to consist of a wave portion (described by the complex traveling wave) and a particle portion consisting of creation and annihilation operators. Quantum field theory mathematically unifies the wave and particle pictures for all matter not just photons or electrons.

The previous few paragraphs point out the importance of quantum optics and some very interesting sources of noise in electromagnetic systems. Although quantum noise is interesting and important, other forms of noise such as RIN and thermal noise must be addressed.