Section 3.10: Dispersion in Waveguides

The rate at which light propagates along a waveguide depends on the frequency of the wave and upon the construction of the waveguide. We discuss intermodal and intramodal dispersion and how they limit the bandwidth of communication systems. The term "mode" appears in a number of contexts. The waveguide mode refers to the particular zig-zag path along which the beam can propagate. This is equivalent to specifying the transverse wave pattern embodied by the "h" wave vectors in previous sections. Alternately, the mode can be specified by the pattern of bright spots observed on an output screen. .

"Dispersion in waveguides" refers to the spreading of a pulse of light as it travels the length of the waveguide. We will use rectangular optical fiber as our prototype waveguide. We consider two basic mechanisms responsible for the spreading. The first concerns the construction of the waveguide. Light can follow a zig-zag path with some paths longer than others. Also, some light penetrates into the material with lower index and therefore travels faster than the light not penetrating as far. The second mechanism concerns the index of refraction. Material dispersion refers to the fact that light with different frequencies (i.e., different colors) travel at differing speeds. Although not considered here, we might expect the speed of light to depend on polarization as well.

We can distinguish between intermodal and intramodal dispersion. *Intermodal* dispersion refers to light, once injected into the fiber, travelling in multiple waveguide modes at the same time. As mentioned above, the different modes have different path lengths and lead to varying penetration into the low index cladding. Therefore, various parts of the wave travel at various speeds and the pulse must broaden. *Intramodal* dispersion refers to light that travels in exactly one waveguide mode (such as for a single mode fiber). In this case, we eliminate any delays due to light propagating in multiple modes (different path lengths for example). However, the waveguide group velocity can still depend on construction. For example, consider light made of multiple frequencies propagating in a single mode. In this case, light with longer wavelength penetrates further into the lower-index cladding and therefore travels faster. Also, the refractive index depends on frequency.

Topic 3.10.1: The Dispersion Diagram

In general, the dispersion diagram displays ω vs k or E vs k, where $E = \hbar\omega$ represents energy. The slope of the curves in the dispersion diagram gives the group velocity of EM waves. This topic applies the same ideas for a wave propagating along the length of the waveguide.

The dispersion diagram for a waveguide shows the relation between the angular frequency ω and the effective propagation constant β. The slope provides the group velocity of the wave along the length of the waveguide. Figure 3.10.1 shows an example (Reference 10, Kasap's book). Some

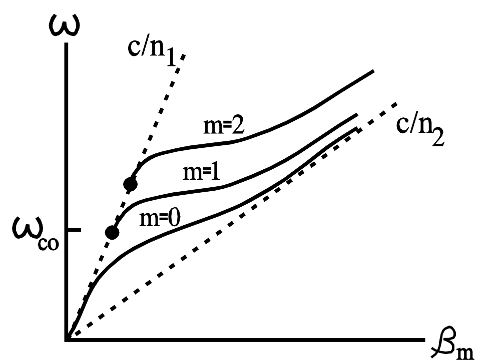

Figure 3.10.1: Dispersion curves for fiber in TE modes (after Kasap).

points should be noted. First, the diagram shows that for a given ω, only certain values of β are allowed (as found in previous sections); these values are found by drawing a horizontal line through the chosen value of ω. Second, if ω (the color) varies continuously so does β for values past cut-off. Third, the two dotted lines give the maximum and minimum waveguide group velocities. Previous sections demonstrate the minimum and maximum values of β according to

$$\beta_{min} = k_o n_1 \le \beta \le k_o n_2 = \beta_{max}$$

where

$$k_o = \frac{2\pi}{\lambda_o} = \frac{\omega}{c}$$

and k_o is the wave vector in vacuum. Therefore, we can find the minimum and maximum waveguide group velocity according to (ignoring any dependence of n on ω)

$$v_{wg}^{(min)} = \frac{\partial\omega}{\partial\beta_{max}} = \left(\partial\beta_{max} / \partial\omega\right)^{-1} = \left(\partial k_o n_2 / \partial\omega\right)^{-1} = \left(\frac{\partial}{\partial\omega}\frac{\omega n_2}{c}\right)^{-1} = \frac{c}{n_2}$$

and similarly for the maximum

$$v_{wg}^{(max)} = \frac{c}{n_1}$$

The maximum and minimum phase velocity serve as fiduciaries. Forth, the different modes have different cutoff frequencies. The m=0 mode propagates for all frequencies. Near cutoff, each of the modes has very large group velocity indicating that the cladding layer carriers the greater portion of the mode. We therefore expect large penetration into the cladding layer. The smallest frequency and largest wavelength occur at cutoff. For large frequencies, the group velocity asymptotically approaches the lower limit of c/n_2. Apparently away from cutoff, the core of the waveguide carries the majority of the mode where the wave travels slowest. Also for fixed ω, the group velocity at the allowed β tends to be larger for higher mode numbers m because of greater penetration into the cladding.

Topic 3.10.2: A Formula for Dispersion

Dispersion causes waves with different frequencies or composed of different waveguide modes to travel at different speeds. This causes the waves to broaden as they travel the length of the waveguide. Figure 3.10.2 shows a pulse that starts fairly narrow but broadens as it travels along. We would find a range of wavelengths in the Fourier decomposition of the pulses. The dispersion measures the amount of "spreading" per unit length of waveguide (or material). The "spread" can either be measured as distance or as a time. For the distance measure, we can write

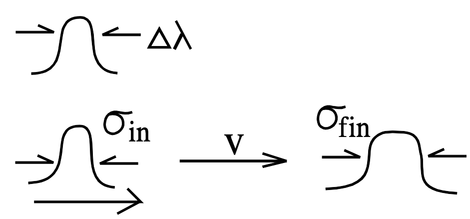

Figure 3.10.2: Pulse spreads as it moves

$$\sigma_{final} - \sigma_{initial} = v\,\Delta\tau$$

where v represents average wave speed, and $\Delta\tau$ denotes the time required to spread from an initial width $\sigma_{initial}$ to the final width σ_{final}. Equivalently, we can say $\Delta\tau$ measures the spreading of the pulse in time. The time method is preferable because it does not require an average velocity.

We can write the dispersion as a formula (dispersions add to first order perturbation theory).

$$\frac{\text{Spread}}{\text{length}} = \frac{\Delta\tau}{\text{length}} = \left(D_m + D_w\right)\Delta\lambda$$

where

$$D_m = -\frac{\lambda}{c}\left(\frac{d^2 n}{d\lambda^2}\right)$$

$$D_w = \frac{1.984 N_{g1}}{\left(\pi t_g\right)^2 2cn_1^2}$$

and where the symbols m and w stand for material and waveguide dispersion, respectively, and N_{g1} represents the group index for material n_1. The group index can be found as follows (the first equality defines N_g).

$$\frac{c}{N_g} = v_g = \left(\frac{\partial k_n}{\partial\omega}\right)^{-1} = \left(\frac{\partial}{\partial\omega}\frac{\omega n}{c}\right)^{-1} = \left(\frac{n}{c} + \frac{\omega}{c}\frac{\partial n}{\partial\omega}\right)^{-1} = \frac{c}{n + \omega\dfrac{\partial n}{\partial\omega}}$$

so that

$$N_g = n + \omega\frac{\partial n}{\partial\omega} = n - \lambda\frac{\partial n}{\partial\lambda}$$

Topic 3.10.3: Bandwidth Limitations

Communications systems have transmitters that modulate a laser and inject the signal into an optical fiber. At the other end of the fiber, a detector circuit receives the signal. Digital transmitters send pulses of light, which represent 0,1. Suppose that R is the repitition rate for the pulses. R has units of #pulses/sec so that the time between a point on one pulse to the identical point on an adjacent one must be $\Delta t = 1/R$. Assume for simplicity that the pulses are very narrow.

The pulse spreads as it moves as shown in Figure 3.10.3. At some point along the fiber, the pulses will start to overlap. We can estimate the maximum possible bit rate B=R by insisting that the pulses remain separated by about $2\Delta\tau_{1/2}$. Therefore we can write

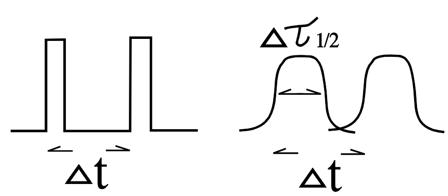

Figure 3.10.3: Spreading pulses limit the bandwidth.

$$B = \frac{1}{\Delta t} = \frac{0.5}{\Delta\tau_{1/2}}$$