Section 2.4: Relations for Cavity Lifetime, Reflectance and Internal Loss

In order to apply the rate equations to "real world" situations, we need to relate the parameters appearing in the rate equation to the physical construction of the device. We can most easily find a relation between the mirror loss and the mirror reflectance. The free carrier absorption and optical scattering for the internal (distributed losses) require the wave equation developed in the next chapter. Here demonstrate the relation for the reflection of optical power without interference effects. We then state the output power for mirrors with significantly unequal reflectance.

Topic 2.4.1: Internal Relations

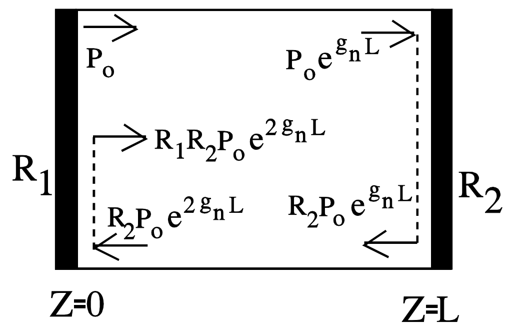

Figure 2.4.1: The effect of the gain medium on the optical power of a beam making a round-trip in the laser Fabry-Perot cavity.

We need to relate the cavity lifetime τ_γ to the reflectance of the two mirrors R_1 and R_2 on the ends of the laser, and to the internal losses such as sidewall scattering and free carrier absorption. The term "reflectance" refers to the amount of optical *power* reflected from a mirror whereas reflectivity usually refers to the elecric field (power is essentially the square of the field). The method presented in this topic uses a laser operating at steady state above threshold and requires the power in the beam to have the same magnitude after a round trip as it did at the start. The procedure again shows that the gain must equal the optical losses above threshold. We should also require the phase of the waves to agree after the round trip, but we will include this effect after discussing the optical scattering and transfer matrices.

We demonstrate the following basic relation for the cavity lifetime

$$\frac{1}{\tau_\gamma} = v_g\alpha_{int} + v_g\alpha_m = v_g\alpha_{int} + \frac{v_g}{2L}\,Ln\left(\frac{1}{R_1R_2}\right) \tag{2.4.1}$$

where R_1 and R_2 denote the reflectivity of the two mirrors and L represents the length of the cavity. Equation 2.4.1 describes the case of the cavity without the gain or absorption normally encountered for semiconductor materials. The equation relates the time required for optical energy to escape from the cavity to the time constants for the internal and mirror loss. The internal loss includes the scattering and free carrier absorption. If the cavity has semiconductor material, we can define an *effective* cavity lifetime τ_{eff} that can be much larger than the cavity lifetime τ_γ since the material can have gain that produces photons thereby compensating for those photons lost. The second term on the right-hand side of Equation 2.4.1 describes the mirror loss. The factor 1/L provides an average loss over the length L of the laser. The logarithm term describes the "fractional loss" at the mirrors. The time required for light to travel from one mirror to the other L/v_g must be the same as the time interval during which the light attempts to escape from one mirror. The factor of 1/2 occurs because the light makes a round trip.

The relation (2.4.1) can be derived by requiring the optical power within the cavity to maintain steady state. This means that a beam starting at z=0 (with power P_o), reflecting from the mirror at z=L, and finally reflecting from the mirror at z=0, must have the same power with which it started. The number of photons starting at z=0 must be the same as the number returning to z=0 after a round trip.

We must calculate the increase and decrease of the optical energy as it propagates from z=0 to z=L and back to z=0. We first consider the exponential growth of the wave as it propagates across the gain medium. Consider photons starting at the left mirror and traveling to the right mirror across the length L of the gain medium. These photons encounter distributed internal loss α_{int} and gain, but not mirror loss. The photon rate equation (2.3.2) without spontaneous recombination can be written

$$\frac{d\gamma}{dt} = +\Gamma v_g g(n)\gamma - \frac{\gamma}{\tau_\gamma} \tag{2.4.2}$$

where the distributive losses produce the cavity lifetime of

$$\frac{1}{\tau_\gamma} = \frac{1}{\tau_{int}} = v_g \alpha_{int} \tag{2.4.3}$$

The mirror-loss term does not appear in Equation 2.4.3 because we first consider an EM wave that only propagates between mirrors. Setting the confinement factor equal to one $\Gamma = 1$ for convenience, and changing variables from time "t" to distance "z", Equations 2.4.2 and 2.4.3 provide

$$\frac{d\gamma}{dt} = v_g \frac{d\gamma}{dz} = v_g g\gamma - \gamma v_g \alpha_{int} \qquad \text{or} \qquad \frac{d\gamma}{dz} = g\gamma - \gamma\alpha_{int} = g_{net}\gamma \tag{2.4.4}$$

where the net gain

$$g_{net} = g - \alpha_{int} \tag{2.4.5}$$

accounts for the material gain and distributed losses. The photon density is proportional to the power P(z) travelling through the medium so that Equation 2.4.4 can also be written as

$$\frac{dP}{dz} = g_{net} P \tag{2.4.6}$$

Assuming the carrier density is constant along z, the solution to this simple differential equation is

$$P(z) = P_o \exp(g_n z) \tag{2.4.7}$$

So the power grows exponentially as it propagates from z=0 to z=L. Just before the right-hand mirror in Figure 2.4.1, the power must be

$$P(L) = P_o \exp(g_n L) \tag{2.4.8}$$

Now calculate the power after a round trip by repeatedly using Equation 2.4.8. The power at z=L for a beam starting at z=0 is

$$P_o \exp(g_n L)$$

The reflectance R of mirror R2 decreases the power to

$$R_2 P_o \exp(g_n L)$$

Reflectance refers to the ratio of the reflected to incident *power*; it is the square of the *reflectivity* $R=r^2$ (reflectivity refers to the fields). The power in the beam increases exponentially as it travels from z=L back to z=0

$$R_2 P_o \exp(2 g_n L)$$

Finally, mirror R1 reduces the beam power to produce the power

$$R_1 R_2 P_o \exp(2 g_n L)$$

just to the right of the mirror at z=0. At this time, the beam has made a complete round trip. For steady state, the initial power P_o must be the same as the final power

$$P_o = R_1 R_2 P_o \exp(2 g_n L)$$

which yields the relation

$$g_{net} = \frac{1}{2L} \operatorname{Ln}\left(\frac{1}{R_1 R_2}\right) \tag{2.4.9a}$$

where

$$g_{net} = g - \alpha_{int} \tag{2.4.9b}$$

The material gain g in Equation 2.4.9b can be rewritten by appealing to Equation 2.3.11 which says that the gain must equal the loss at steady state $\Gamma g = \alpha$ or, for unity confinement $\Gamma = 1$, we have $g = \alpha$. We can also appeal to the equivalent form in terms of the cavity lifetime and the temporal gain g_t

$$v_g g = g_t = \frac{1}{\tau_\gamma} \tag{2.4.9c}$$

Combining Equations 2.4.9 provides the cavity lifetime (without the gain or absorption due to semiconductor material)

$$\frac{1}{\tau_\gamma} = v_g \alpha_{int} + \frac{v_g}{2L} \operatorname{Ln}\left(\frac{1}{R_1 R_2}\right) \tag{2.4.10}$$

This last equation provides the lifetime for the case when energy escapes from the cavity through "internal loss" and through *both* mirrors. If the two mirrors have the same reflectivity (as is typical for a semiconductor laser with cleaved or etched facets), then $R=R_1=R_2$ and

$$\frac{1}{\tau_\gamma} = v_g \alpha_{int} + \frac{v_g}{L} \operatorname{Ln}\left(\frac{1}{R}\right) \tag{2.4.11}$$

The power reflectivity for facets cleaved in GaAs is nearly 0.34. Typically $\alpha_m \cong 100\ cm^{-1}$ and $\alpha_{int} \cong 50\ cm^{-1}$.

Topic 2.4.2: External Relations

In this topic, we illustrate the output power from mirrors with reflectance R_1 and R_2. Often times one mirror receives a high reflectance coating to increase the power out of the other one. The photon density (intracavity power) varies significantly from a constant value near the mirrors. A higher reflectance mirror produces a higher photon density just inside that cavity than does a lower reflectance one. By calculating the average photon density in the laser body (Refence 5, the Agrawal and Dutta book) and applying the appropriate boundary conditions, we find the power P_1 and P_2 through the mirrors with reflectance R_1 and R_2, respectively,

$$P_1 = \frac{(1-R_1)\sqrt{R_2}}{\left(\sqrt{R_1}+\sqrt{R_2}\right)\left(1-\sqrt{R_1 R_2}\right)} P_o \tag{2.4.12a}$$

$$P_2 = \frac{(1-R_2)\sqrt{R_1}}{\left(\sqrt{R_1}+\sqrt{R_2}\right)\left(1-\sqrt{R_1 R_2}\right)} P_o \tag{2.4.12b}$$

where P_o gives the total power through *both* mirrors given in Equation 2.3.19. The relation between the reflectance and the loss coefficients is given by Equation 2.4.11. We can easily show that $P_1 + P_2 = P_o$, which indicates the output power divides itself between the two facets. If the facets are identical $R_1 = R_2 = R$ then each facet handles half the total since then $P_1 = P_2 = P_o / 2$.